

Ending Elementary Errors of the British Maths Era

1522 – 2022

INTERNATIONAL CONFERENCE ON HISTORY OF MATHEMATICS

CENTRE FOR INDIAN KNOWLEDGE SYSTEMS, IIT MADRAS

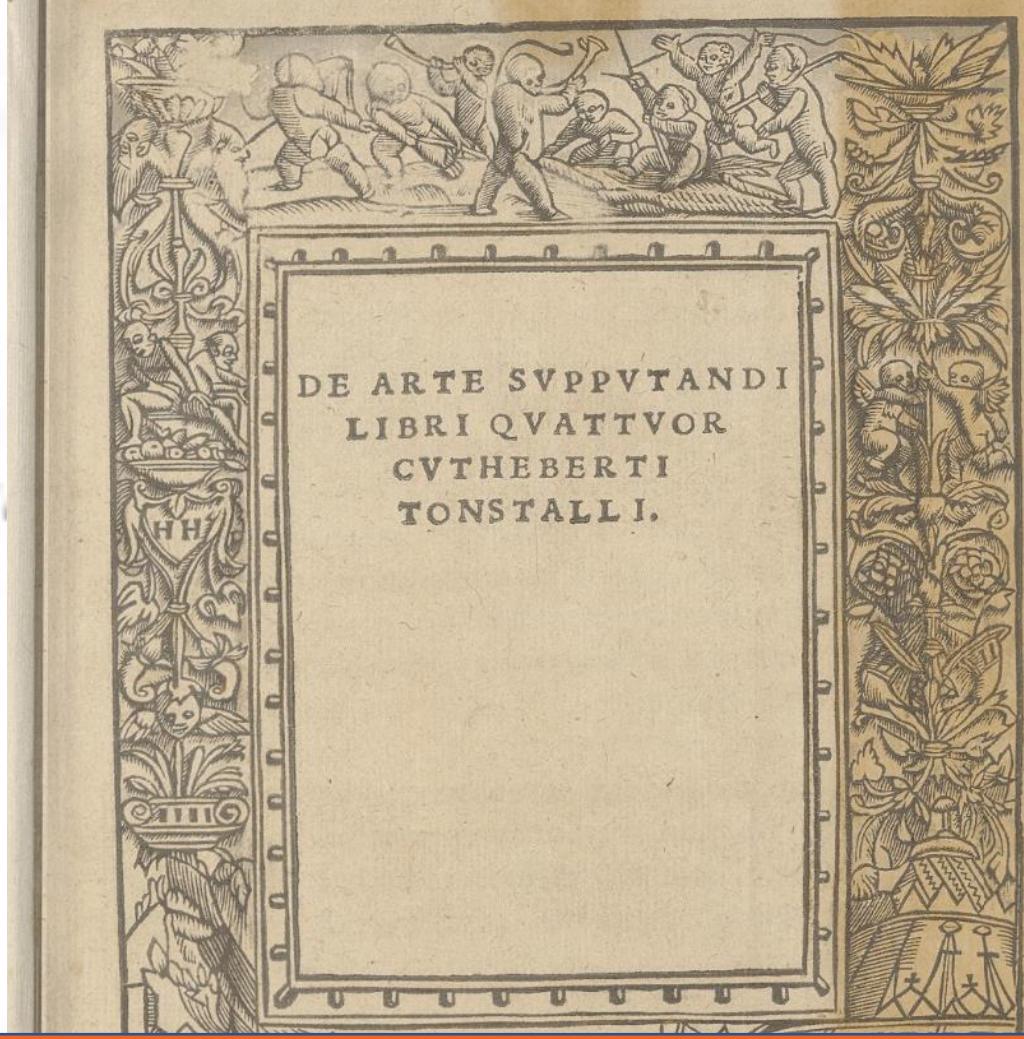
in association with

INDIAN SOCIETY FOR HISTORY OF MATHEMATICS

Jonathan J. Crabtree

www.jonathancrabtree.com

www.podometric.in



DE ARTE SUPPUTANDI LIBRI QUATTUOR
The Art of Calculation in Four Books
By Catholic Priest Cuthbert Tunstall 1522



De Additione.

$$\begin{array}{r} 69001303 \\ 69000000 \\ 69005000 \\ 790\cancel{8}7100 \\ 790000003 \\ 79006000 \\ 89004000 \\ 89026100 \\ 89008000 \\ 99005000 \\ 99002400 \\ 99017001 \\ \hline . & . \\ 1008095008 \end{array}$$

A week later King Henry VIII made Tunstall Bishop of London. Tunstall was at the end of a series of ideas copied from others...

1494

- **Luca Pacioli**
- Summa de arithmeticā, geometriā,
proportioni et proportionalitā

c.1475

- **Piero della Francesca**
- Trattato d'abaco

1202

- **Leonardo Pisano**
- Liber Abaci

1494

- Luca Pacioli
- Summa de
arithmetica,
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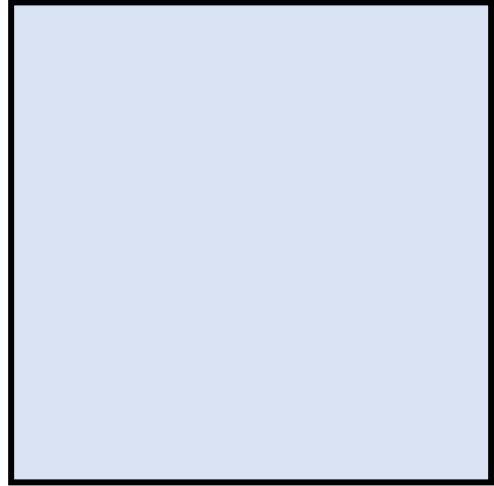
Luca Pacioli on $(10 - 2) \times (10 - 2)$

NOTE Below the \tilde{m} means today's – symbol

“10 \tilde{m} 2 equals 8; this means that if 10 \tilde{m} 2 is multiplied by 10 \tilde{m} 2 the result is 64; if however, the cross multiplication is applied, we obtain 10 multiplied by 10, namely 100, then 10 twice multiplied by \tilde{m} 2, which gives \tilde{m} 40, which together give 60; thus it becomes evident that \tilde{m} 2 multiplied by \tilde{m} 2 should give the number 4.”

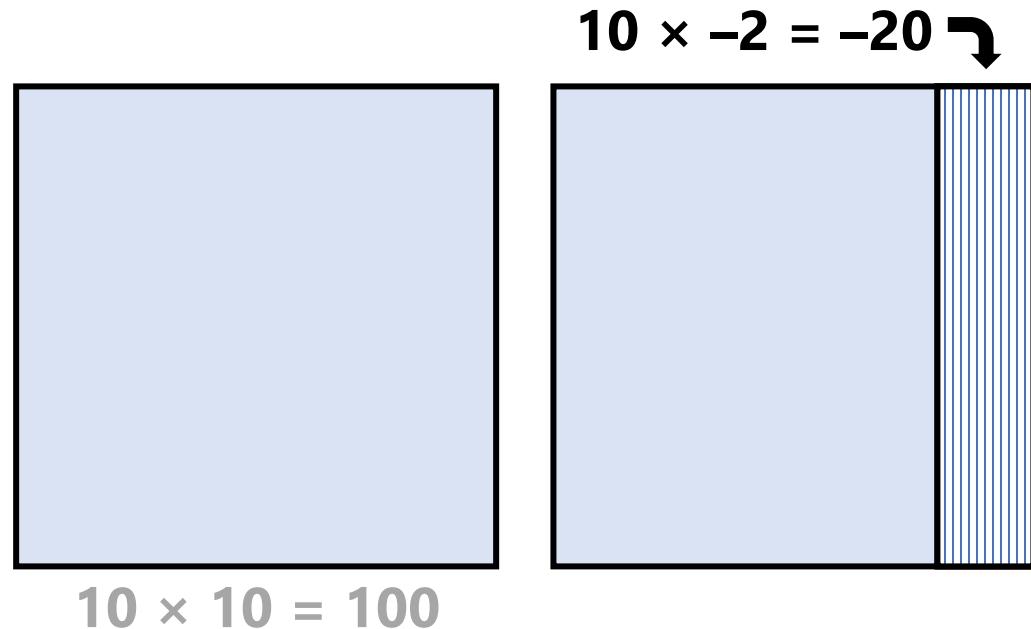
Juschkewitsch, A. P.: 1964, *Geschichte der Mathematik in Mittelalter*, übersetzt von V. Ziegler. Basel: Pfalz-Verlag. Trans. In Thomaidis Y. (1993). *Aspects of negative numbers in the early 17th century : an approach for didactic reasons.* Science & Education : Contributions from History Philosophy and Sociology of Science and Mathematics 69–86.

Luca Pacioli on $(10 - 2) \times (10 - 2)$

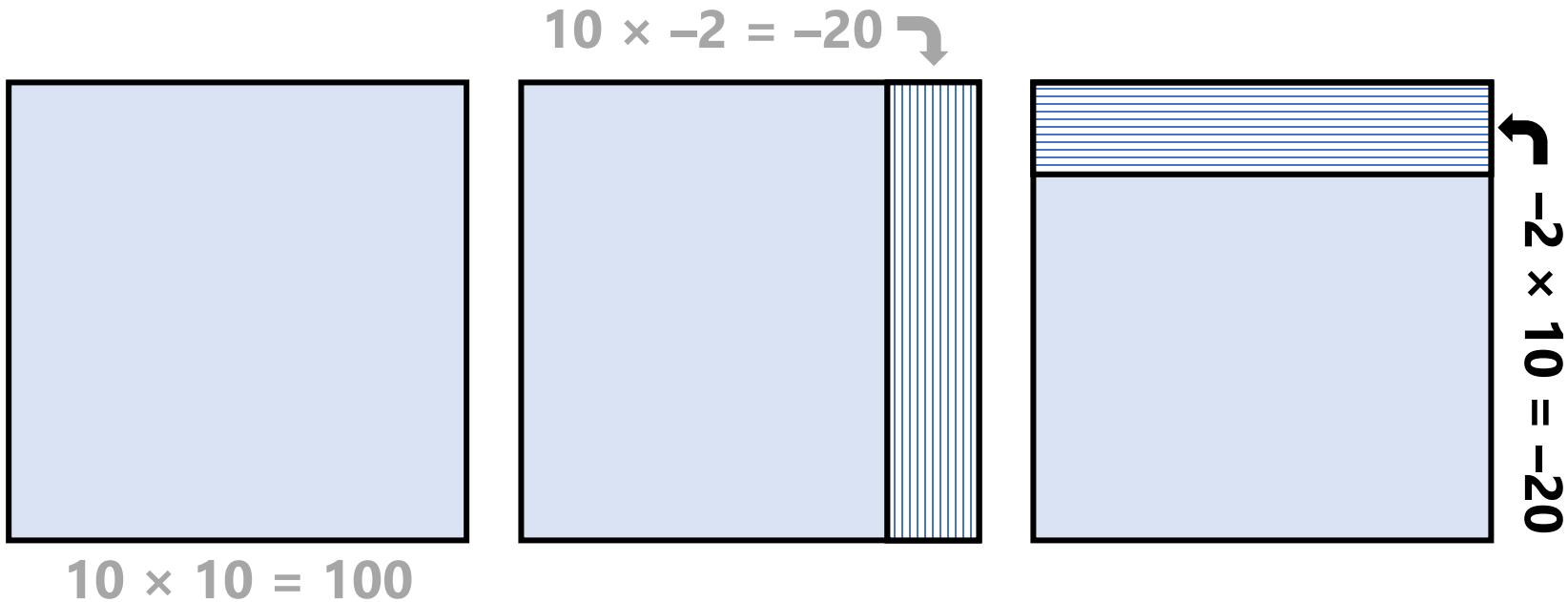


$$10 \times 10 = 100$$

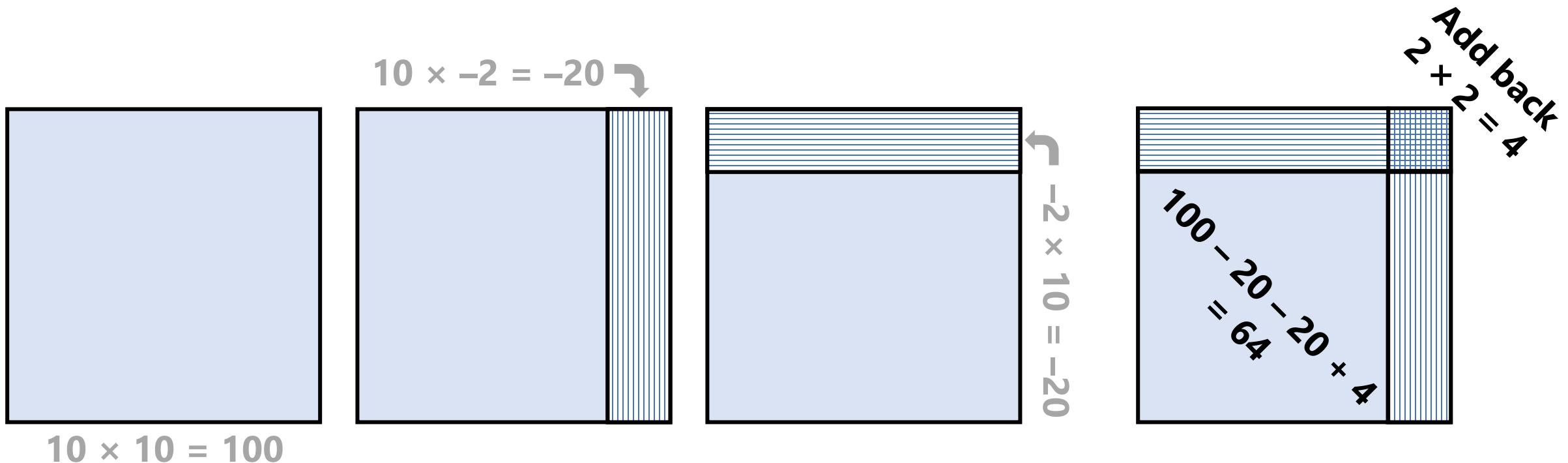
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Luca Pacioli on $(10 - 2) \times (10 - 2)$

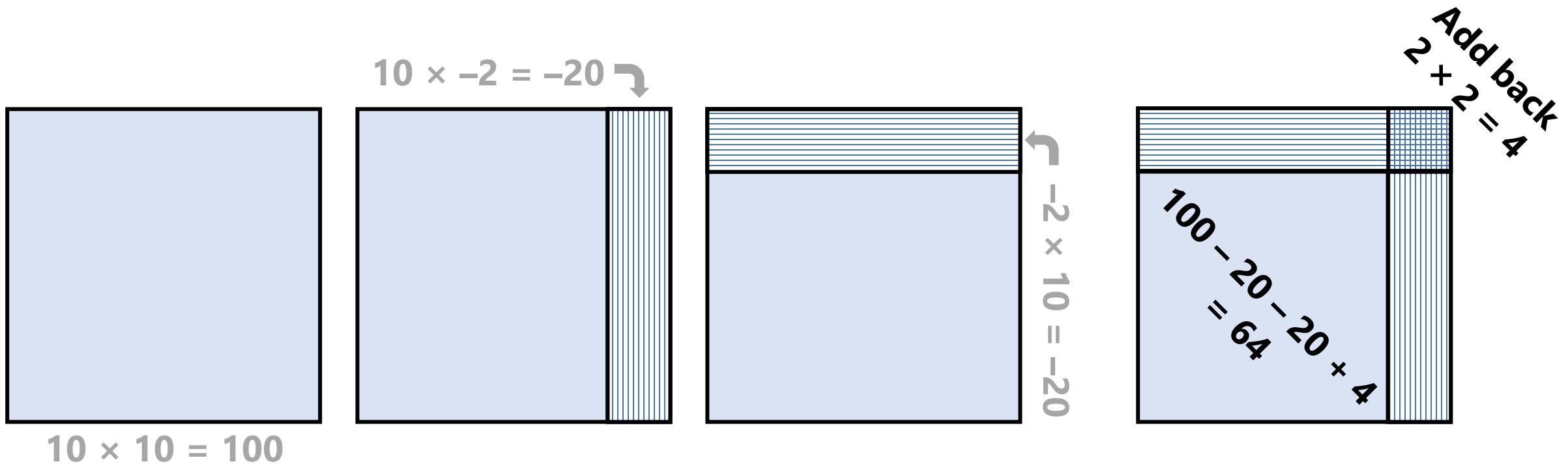


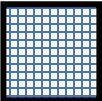
Luca Pacioli on $(10 - 2) \times (10 - 2)$



- **The top right corner measuring 2×2 was removed twice instead of once, so it must be added back once!**

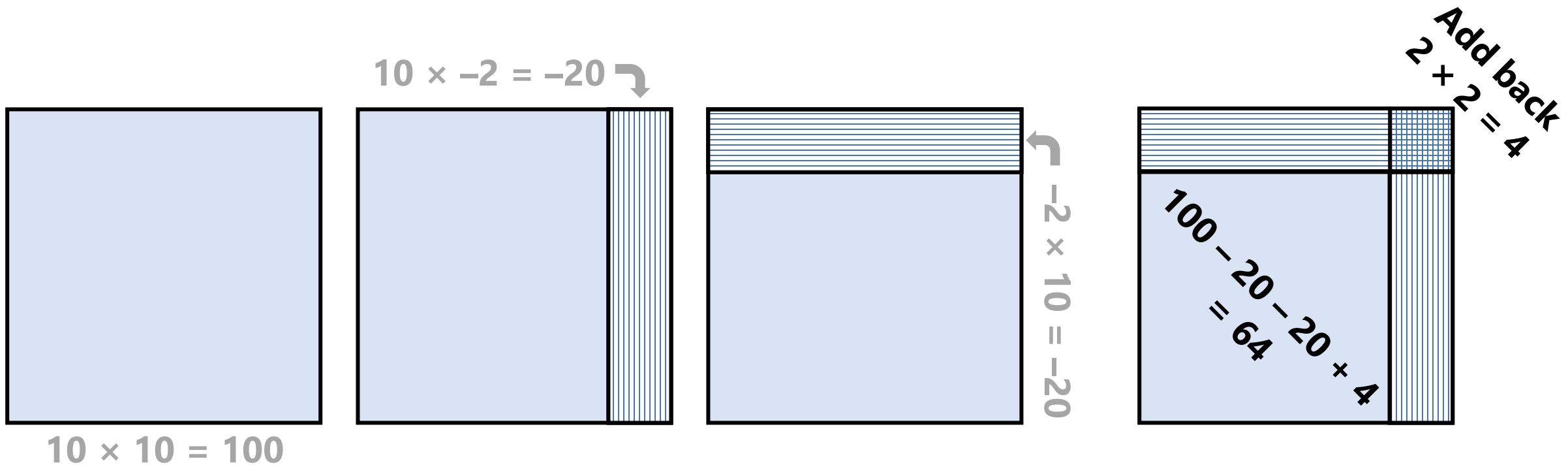
Luca Pacioli on $(10 - 2) \times (10 - 2)$

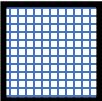


 **The top right corner measuring 2×2 was removed twice instead of once, so it must be added back once!**

**There are no negative numbers in $(10 - 2) \times (10 - 2)$.
There are just positive numbers being subtracted.**

Luca Pacioli on $(10 - 2) \times (10 - 2)$



-  **The top right corner measuring 2×2 was removed twice instead of once, so it must be added back once!**

**There are no negative numbers in $(10 - 2) \times (10 - 2)$.
There are just positive numbers being subtracted.**

For -8×-8 we would need $(2 - 10) \times (2 - 10)$.

1494

- **Luca Pacioli**
- Summa de arithmeticā, geometriā,
proportioni et proportionalitā

c.1475

- **Piero della Francesca**
- Trattato d'abaco

1202

- **Leonardo Pisano**
- Liber Abaci

c. 825

- **Al-Khwārizmī**
- Al-Khwārizmī's *Algoritmi de numero Indorum* (The Hindu Art of Reckoning)

628

- **Brahmagupta**
- *Brāhmasphuṭasiddhānta*

C. 825

- **Al-Khwārizmī**
- Al-Khwārizmī's *Algoritmi de numero Indorum* (Latin translation of the Arabic)

The Arabic world did not understand India's Zero!
It was defined as a sum of equal and opposite quantities.

628

- **Brahmagupta**
- *Brāhmasphuṭasiddhānta*

THE BASIC ADDITION SUTRAS OF BRAHMAGUPTA

AS1 positive plus positive is positive

AS2 negative plus negative is negative

AS3 positive plus negative is the difference between the positive and negative

AS4 when positive and negative are equal the sum is zero

positive plus zero is positive

AS5 negative plus zero is negative
zero plus zero is zero

THE BASIC ADDITION SUTRAS OF BRAHMAGUPTA

AS4 when positive and negative are equal the sum is zero

NONE OF THESE ARABIC AUTHORS ON INDIAN MATHS UNDERSTOOD INDIA'S LAWS OF ZERO

Al-Khwārazmī

Al-Nīsābūrī

Al-Uqlīdisī

Kūshyār ibn Labbān

Al-Baghdādī

Al-Şardafī

Al-Samaw'al

Ibn al Yāsamīn

Al-Hasṣār

Ibn Mun'im

Nasīr al-Dīn al-Tusī

Al-Abharī

Ibn al-Bannā

Al-Hawārī

Al-Mawāhidī

Ibn al-Qunfūdh

Ibn al-Hā'im

Al-Kāshī

Ibn al-Majdī

Al-Qalasādi

Rađī al-Dīn ibn al-Ḥanbalī

Al-Ghazzī

Taqī al-Dīn

Yaḥyā al-Ru'aynī

Al-Sakhāwī

Al-Qabāqibī

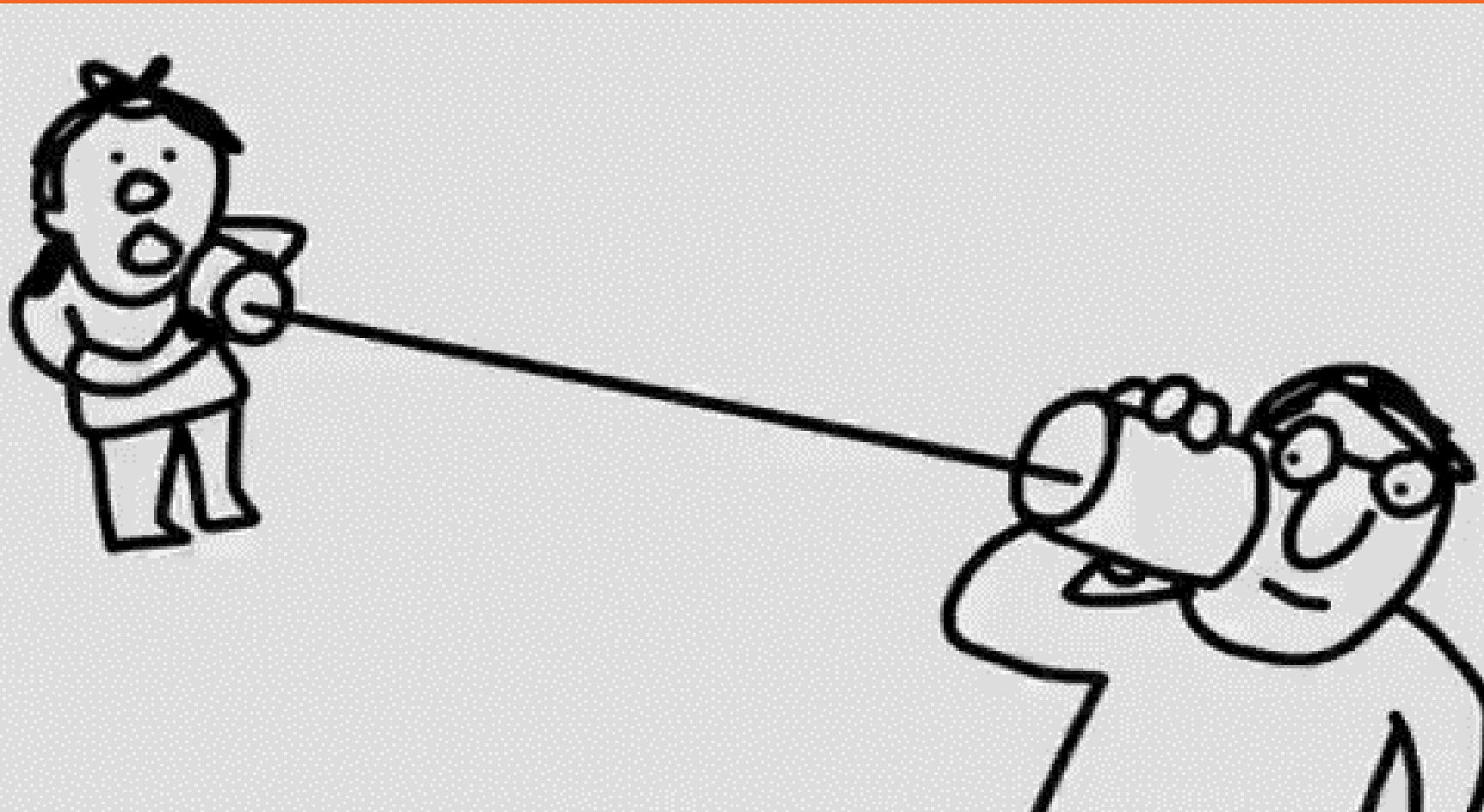
Al-'Āmilī

Nūr al-Dīn al-Anṣārī

Ḥusayn al-Mahallī

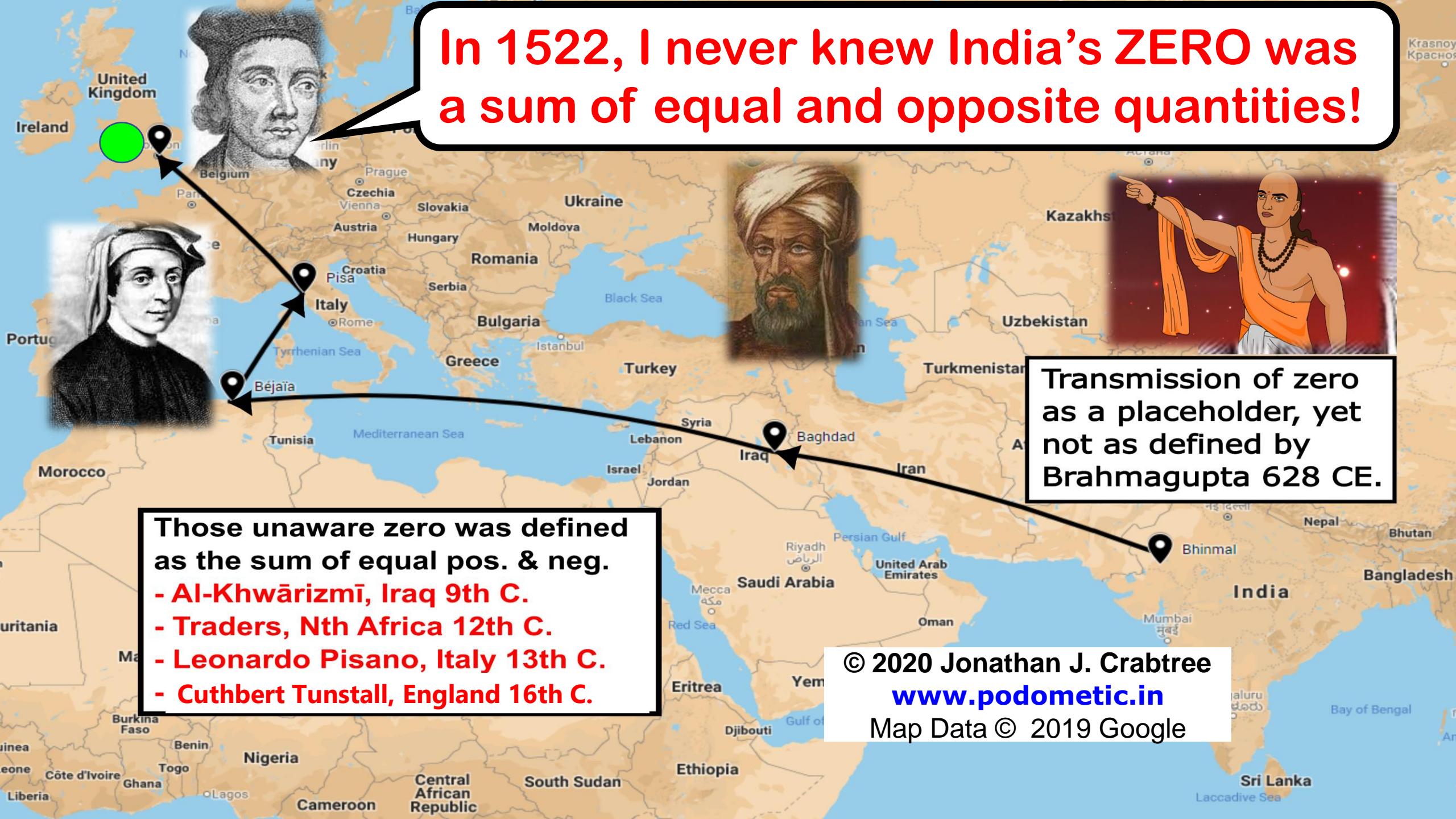
and others

LIKE THE GAME BROKEN TELEPHONE, INDIA'S SYMMETRIC ZERO GOT LOST IN TRANSMISSION





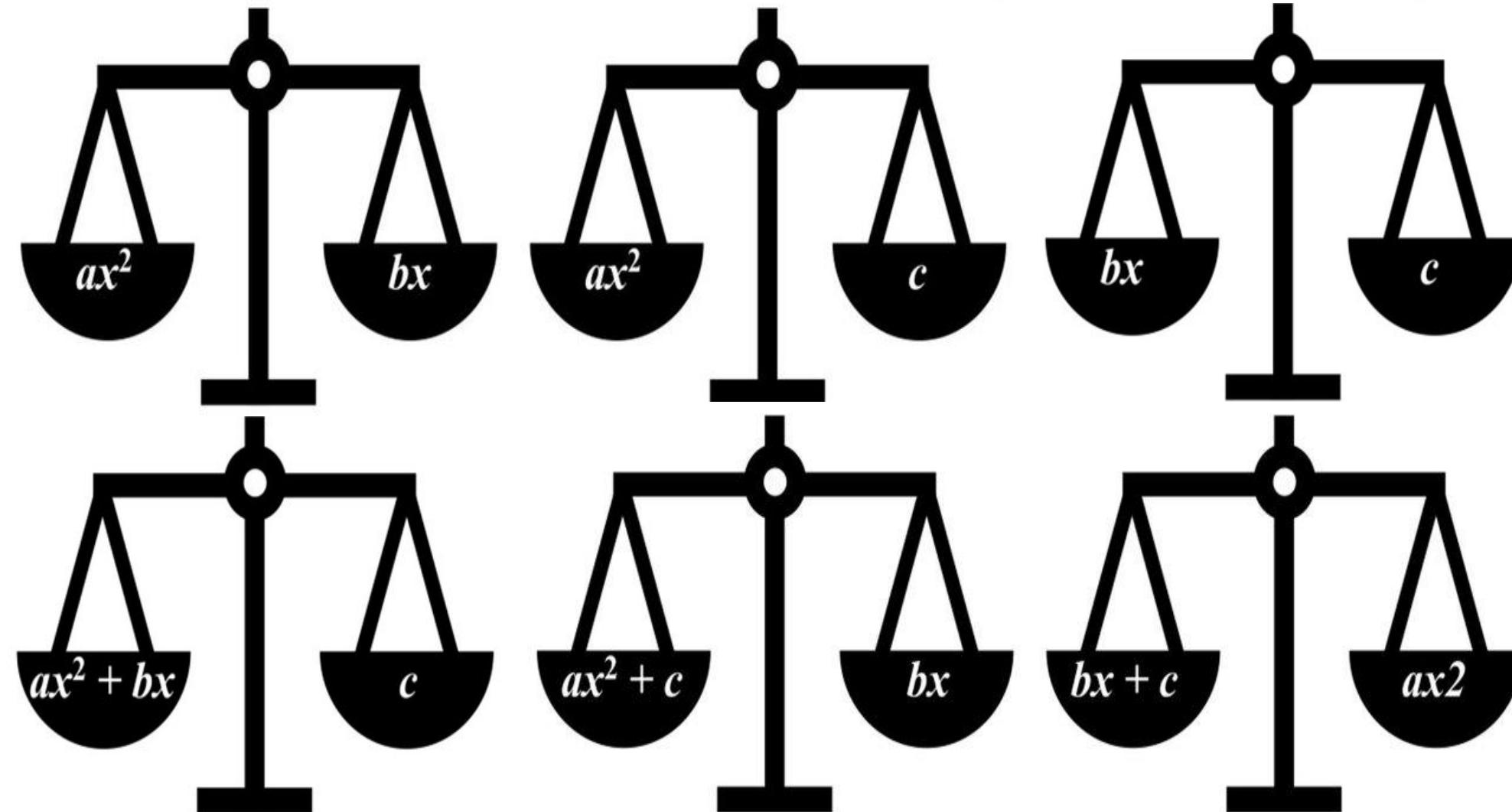
In 1522, I never knew India's ZERO was
a sum of equal and opposite quantities!



NO ZERO OR NEGATIVES IN ARABIC ALGEBRA

Never $ax^2 + bx + c = 0$ or $ax^2 - bx = -c$

Al-Khwārizmī's six types of balanced equations

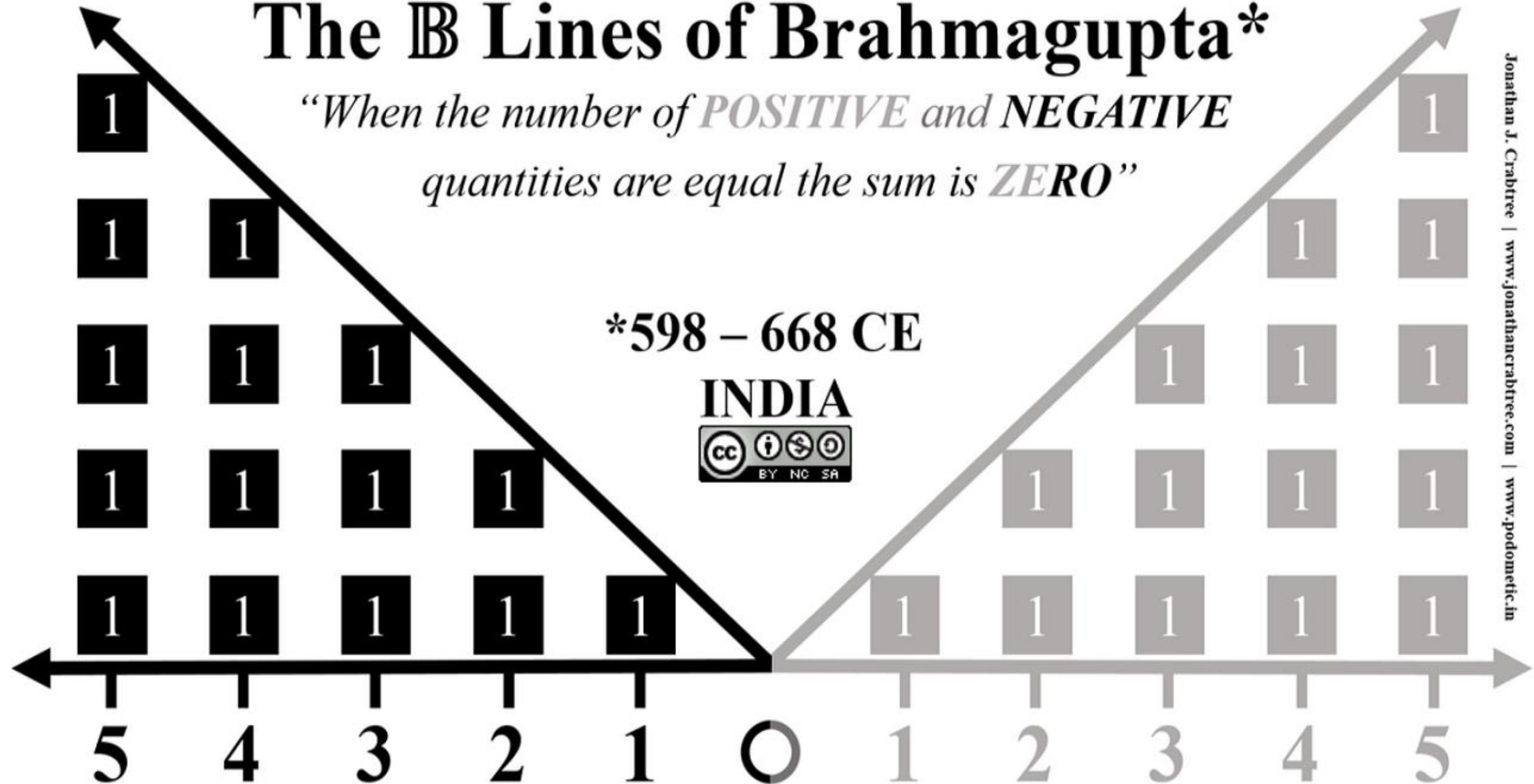


The B Lines of Brahmagupta*

*"When the number of **POSITIVE** and **NEGATIVE** quantities are equal the sum is **ZERO**"*

*598 – 668 CE

INDIA

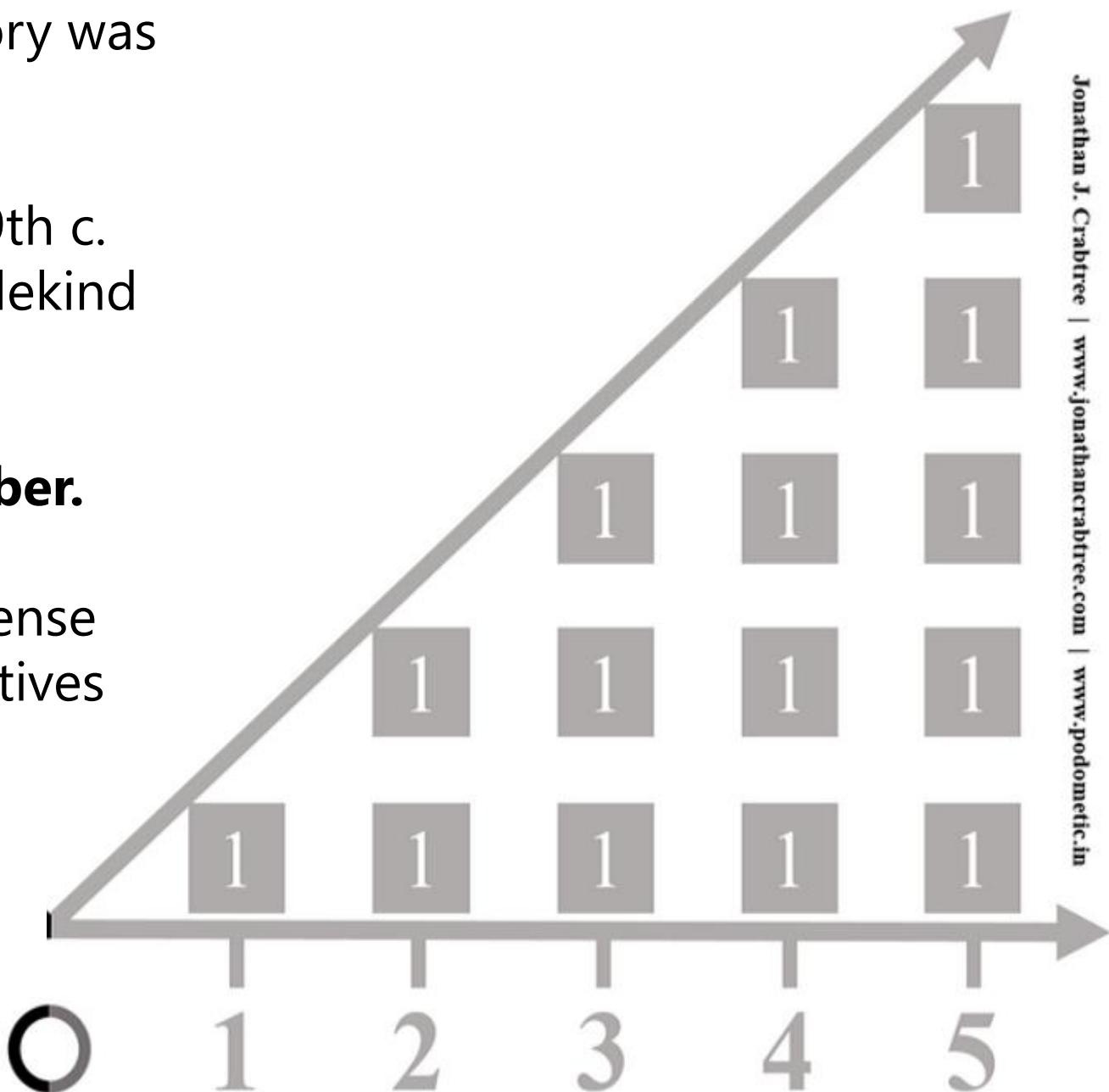


The entire body of western number theory was built upon HALF the set of Integers.

AXIOMS for ARITHMETIC arose in the 19th c.
for NUMBER THEORY via Grassman, Dedekind & Peano.

E.g. 1 is not the successor of any number.

Thus, number theory supports the nonsense notion 5 negatives are LESS than 2 negatives



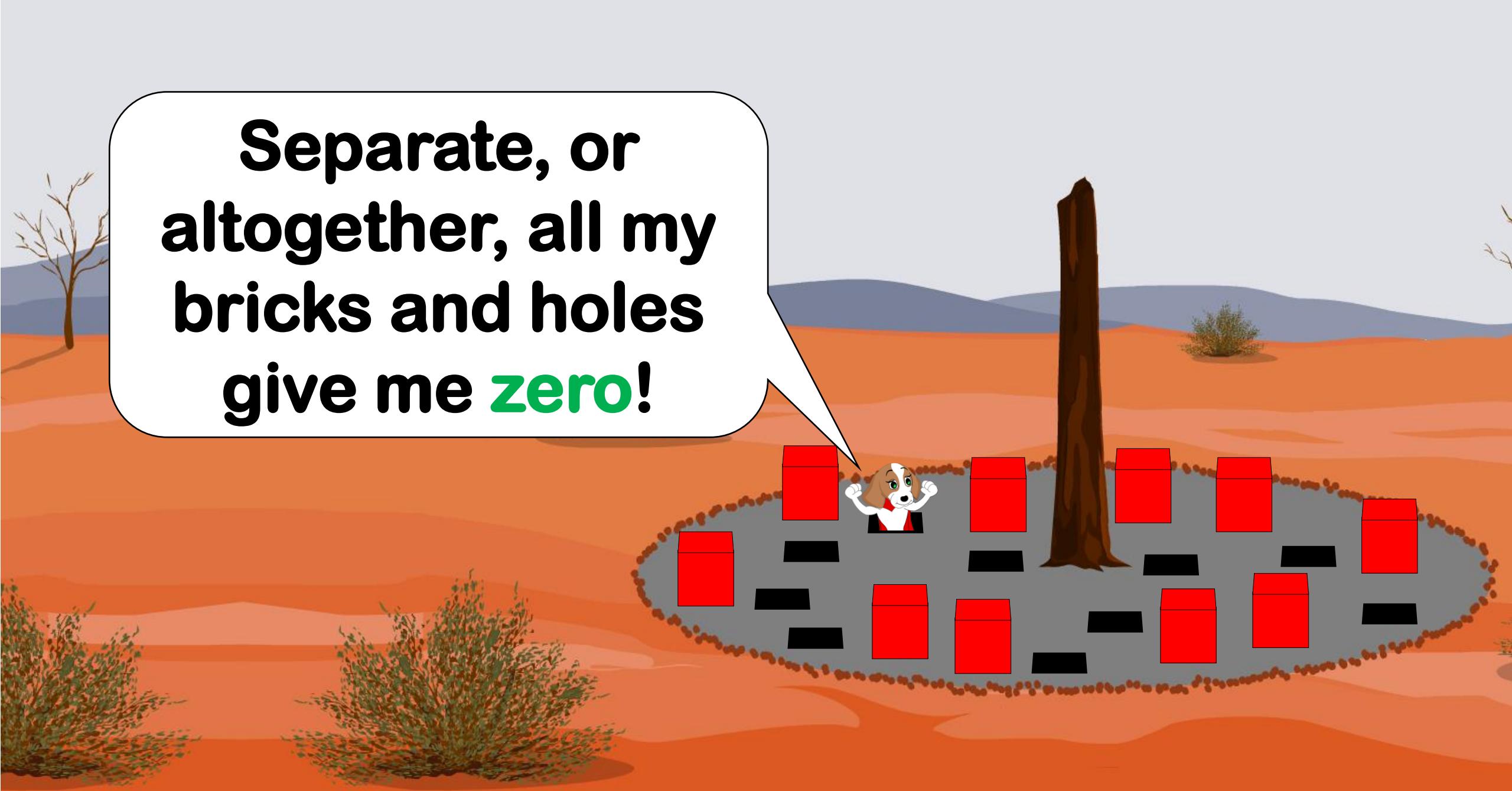
Let's connect the dots behind broken British maths pedagogies of 2022

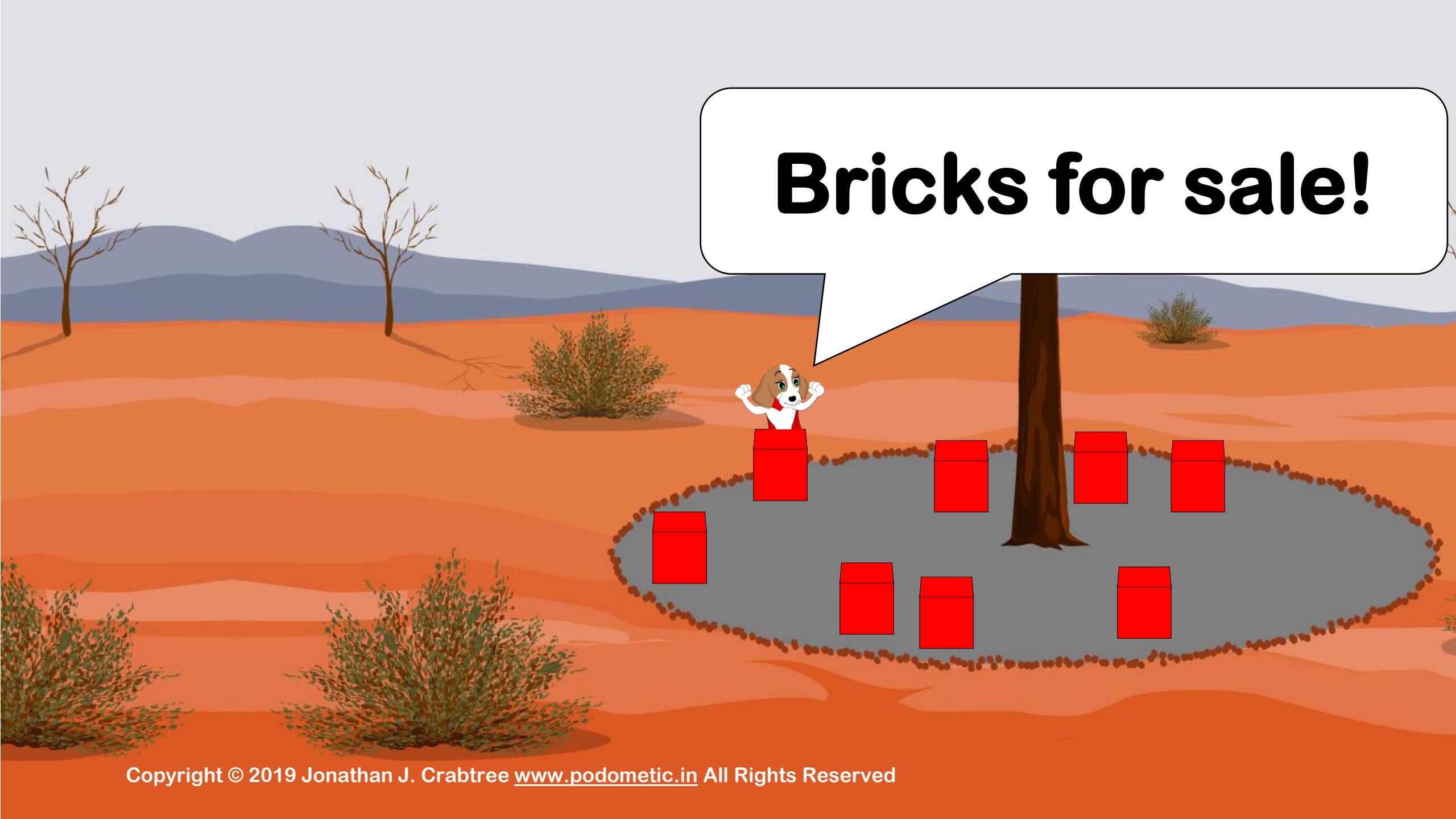
- **Al-Khwarizmi misunderstood India's maths of zero negatives and positives. So...**
- **Leonardo Pisano** misunderstood India's maths of zero negatives and positives. So...
- **Piero della Francesca** misunderstood India's maths of zero negatives and positives. So...
- **Luca Pacioli** misunderstood India's maths of zero negatives and positives. So...
- **Cuthbert Tunstall** misunderstood India's maths of zero negatives and positives. **So in 2022...**

BASIC ALGORITHMIC PROCEDURES FOR CLASS 2 SUBTRACTION ARE MISSING!


$$\begin{array}{r} 08 \\ - 19 \\ \hline \end{array}$$

**Separate, or
altogether, all my
bricks and holes
give me zero!**





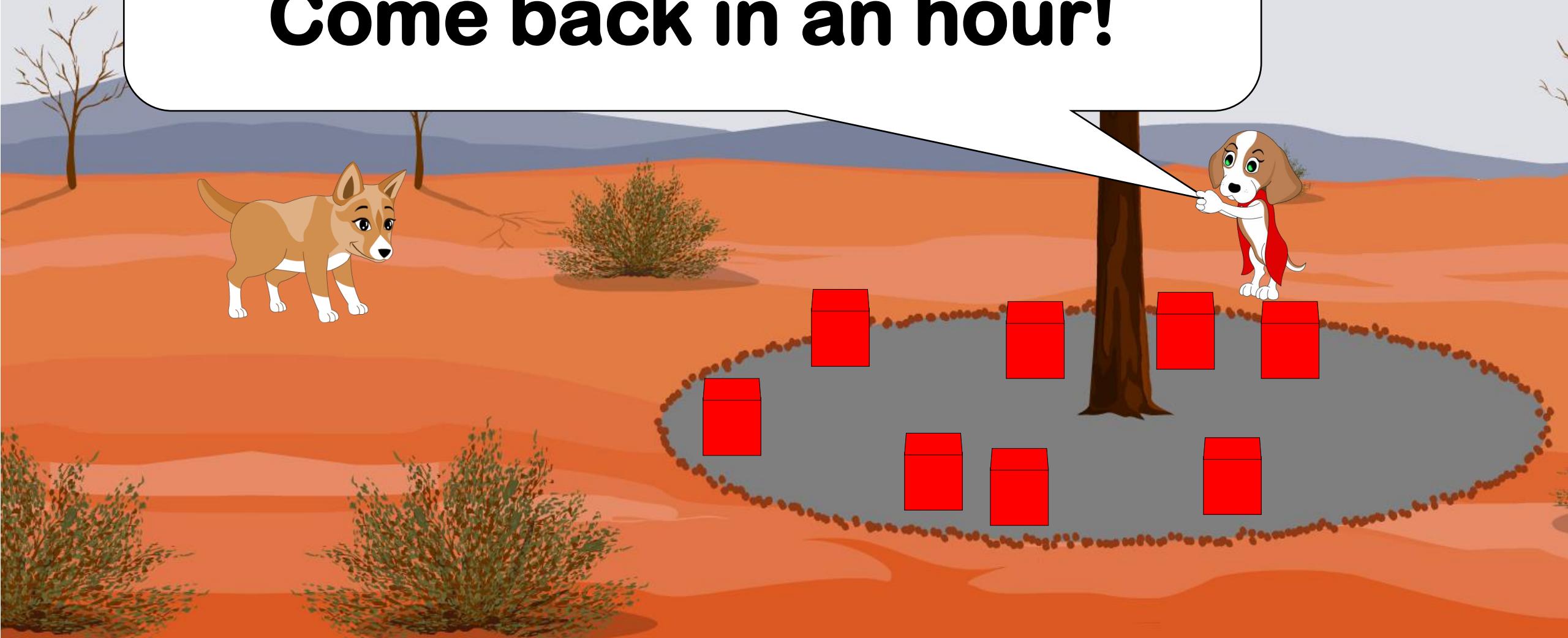
Bricks for sale!



**I want to buy
19 bricks!**

**I only have 8
bricks now...**

**So I need 11 more bricks.
Come back in an hour!**

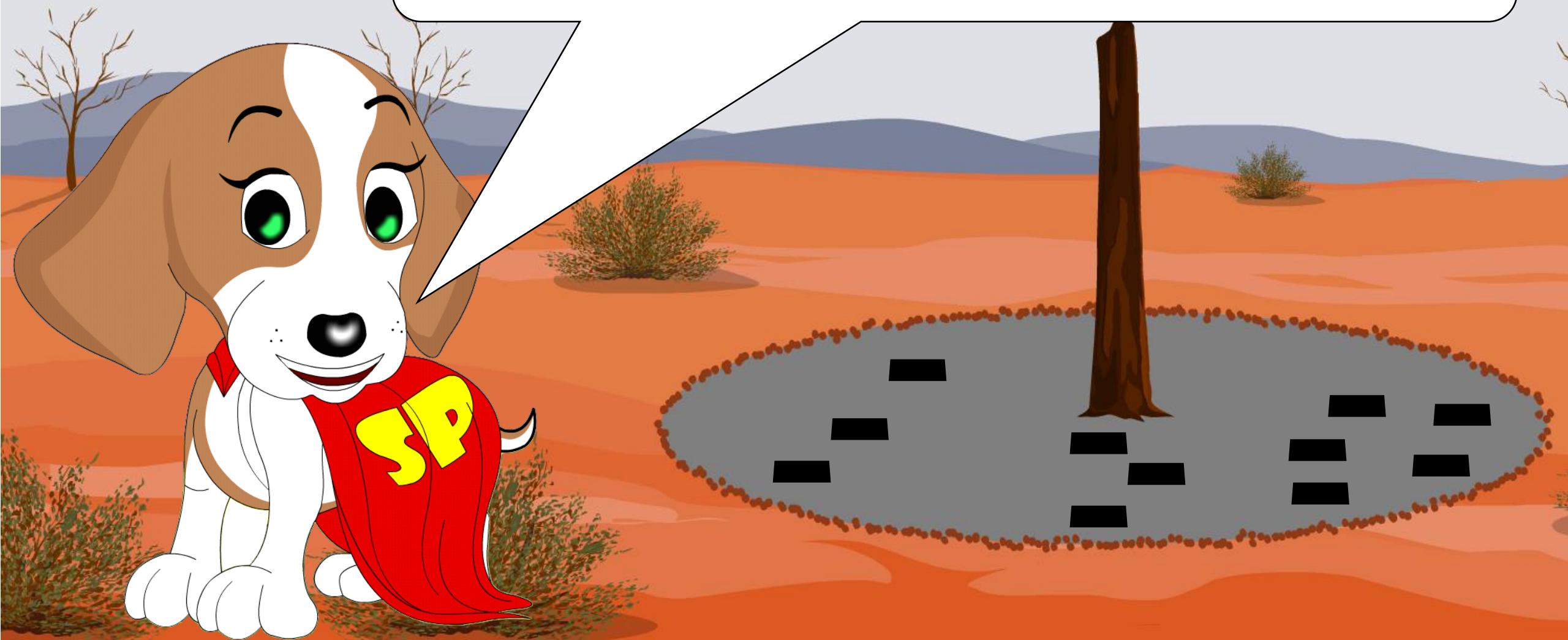


I split ground level zero to dig 11 holes to make another 11 bricks. So you can take away your 19 bricks now.

**Thanks!
I will take
them later.**



Now I have 11 holes for sale!



BASIC ALGORITHMIC PROCEDURES FOR CLASS 2 SUBTRACTION ARE MISSING!

AS5 positive plus zero is positive
negative plus zero is negative
zero plus zero is zero

BASIC ALGORITHMIC PROCEDURES FOR CLASS 2 SUBTRACTION ARE MISSING!

AS5 positive plus zero is positive
negative plus zero is negative
zero plus zero is zero

$$+8 - +19$$

BASIC ALGORITHMIC PROCEDURES FOR CLASS 2 SUBTRACTION ARE MISSING!

AS5 positive plus zero is positive
negative plus zero is negative
zero plus zero is zero

$$-11 + +11 + +8 - +19$$

BASIC ALGORITHMIC PROCEDURES FOR CLASS 2 SUBTRACTION ARE MISSING!

AS5 positive plus zero is positive
negative plus zero is negative
zero plus zero is zero

$$-11 + +11 + +8 - +19$$

$$-11$$

BASIC ALGORITHMIC PROCEDURES FOR CLASS 2 SUBTRACTION ARE MISSING!

SS1 A smaller **positive** subtracted from a larger **positive** **is positive.**

SS2 A smaller **negative** subtracted from a larger **negative** **is negative.**

SS3 If a larger **negative** or **positive** is to be subtracted from a smaller **negative** or **positive**, the sign of their difference is reversed – **negative becomes positive and positive negative.**

SS4 A **negative** minus **zero** **is negative**,
a **positive** minus **zero** **is positive**,
zero minus **zero** **is zero.**

SS5 When a **positive** is to be subtracted from a **negative** or a **negative** from a **positive**, then it is to be added.

BASIC ALGORITHMIC PROCEDURES FOR CLASS 2 SUBTRACTION ARE MISSING!

SS3

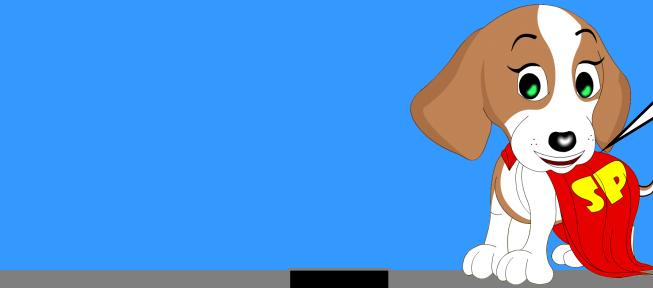
If a larger **negative** or **positive** is to be subtracted from a smaller **negative** or **positive**, the sign of their difference is reversed – **negative becomes positive** and **positive negative**.

7 bricks minus 10 bricks



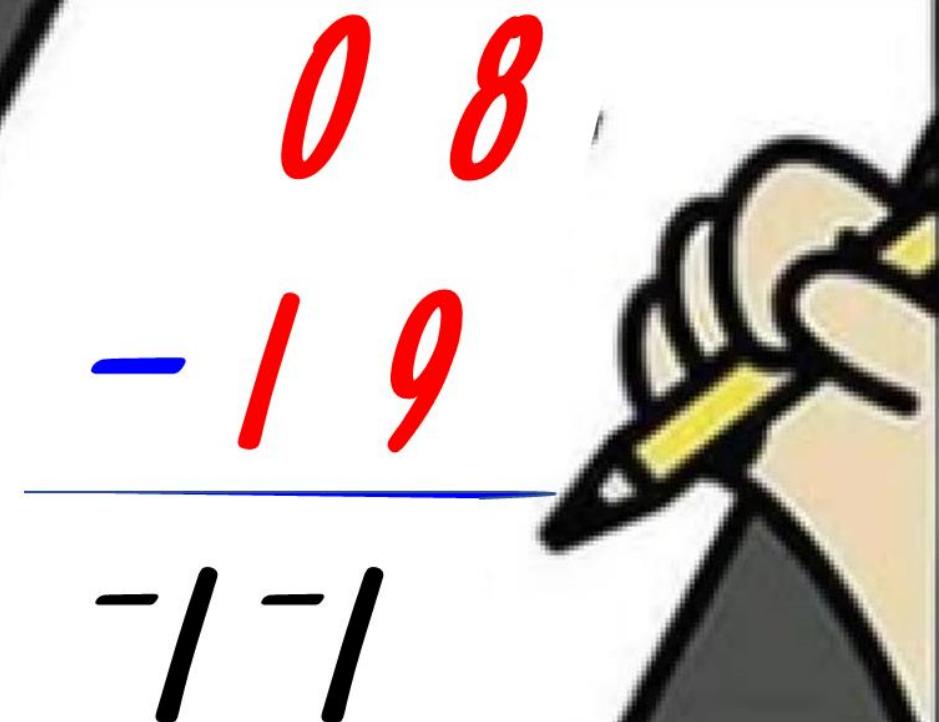
**7 bricks minus 10 bricks
leaves 3 holes remaining**

**When greater is
taken away
from lesser my
things change!**

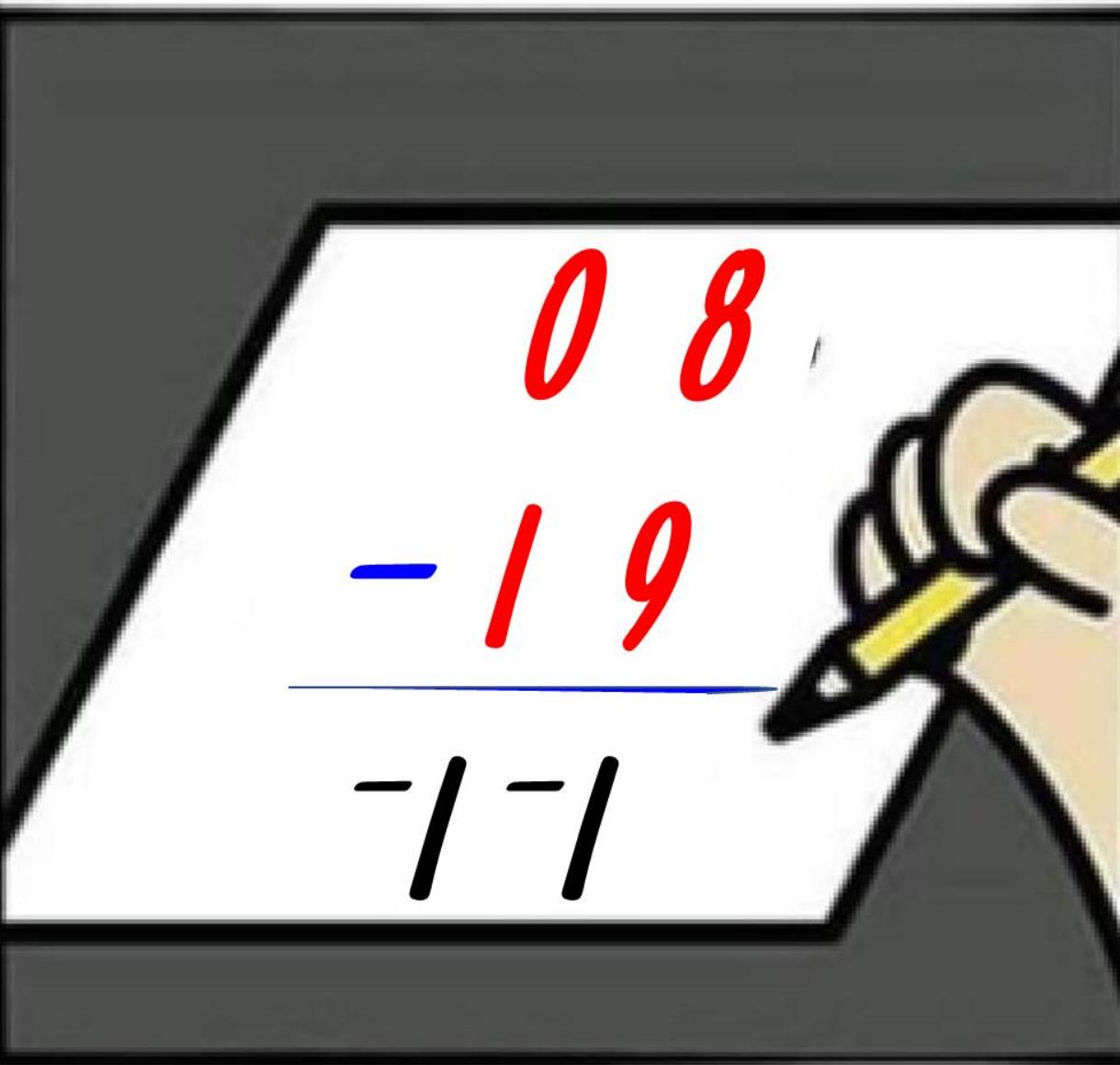


3 holes

BASIC ALGORITHMIC PROCEDURES FOR CLASS 2 SUBTRACTION ARE MISSING!



BASIC ALGORITHMIC PROCEDURES FOR CLASS 2 SUBTRACTION ARE MISSING!



UNITS

$$\begin{array}{r} +8 \\ - +9 \\ \hline -1 \end{array}$$

TENS

$$\begin{array}{r} 0 \\ - +1 \\ \hline -1 \end{array}$$

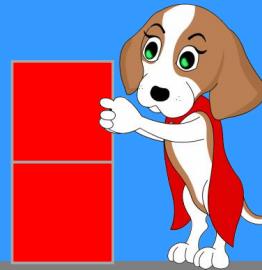
$$\text{So, } +8 - +19 = -11$$

BASIC ALGORITHMIC PROCEDURES FOR CLASS 2 SUBTRACTION ARE MISSING!


$$\begin{array}{r} 13 \\ - 21 \\ \hline -1 +2 \end{array}$$

**Next step, combine
10 holes and 2 bricks**

-1 +2



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**Next step, combine
10 holes and 2 bricks**

-7 +2



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**Next step, combine
10 holes and 2 bricks**

-7 +2



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**10 holes and 2 bricks
make 8 holes**

-8



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Diophantus

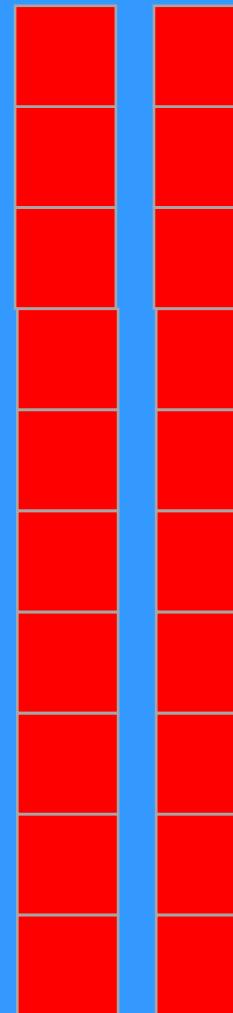
$$4x + 20 = 4$$



Diophantus

$$4x + 20 = 4$$

I need 4 same size things
($4x$) to make 16 bricks go
away leaving 4 bricks. I
need 4 holes!

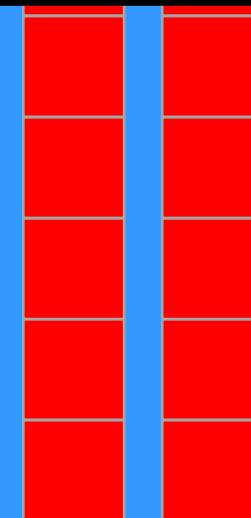


4 things + 20 bricks = 4 bricks

2 things + 10 bricks = 2 bricks

1 thing + 5 bricks = 1 brick

4 holes + 5 bricks = 1 brick



Diophantus

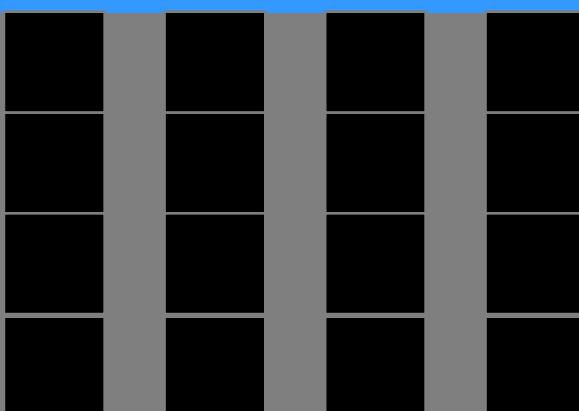
$$4x + 20 = 4$$

With 20 bricks I need 4 holes each 4 deep to make 16 bricks go away to leave 4 bricks.



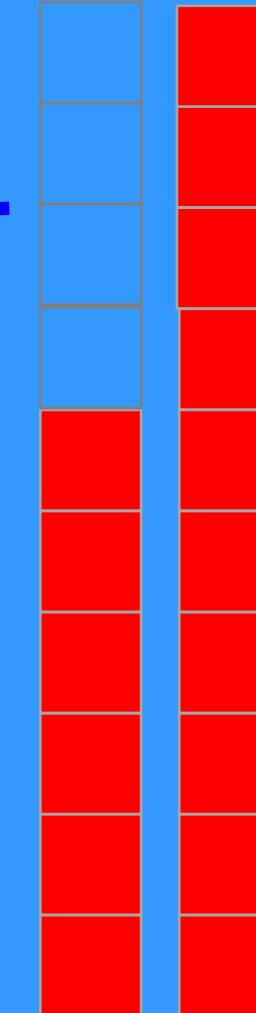
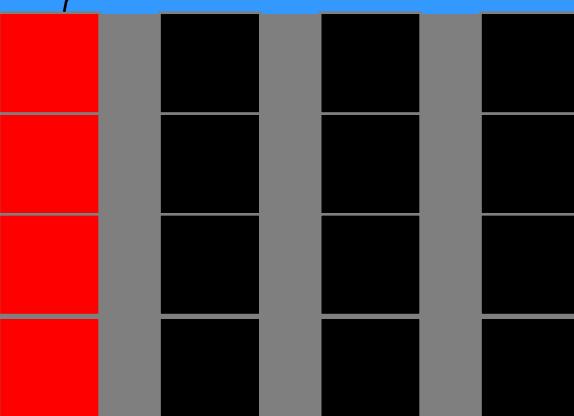
Diophantus

$$4x + 20 = 4$$



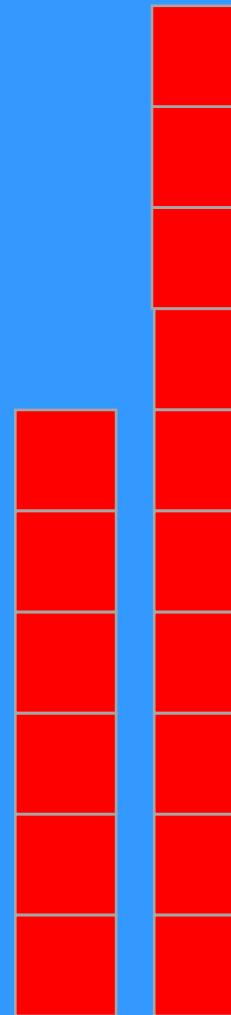
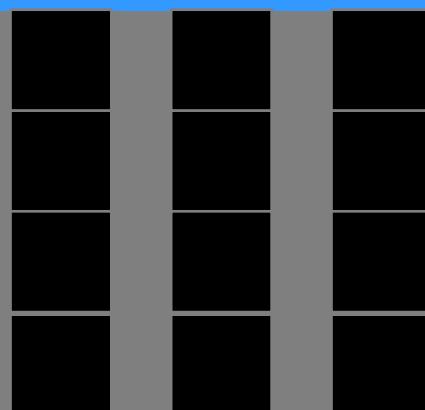
Diophantus

$$4x + 20 = 4$$



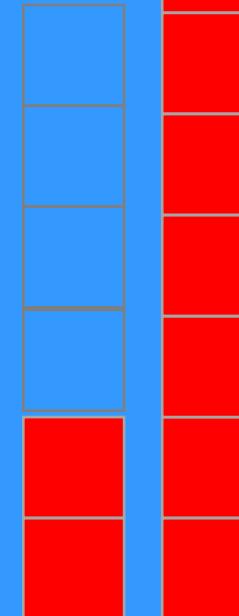
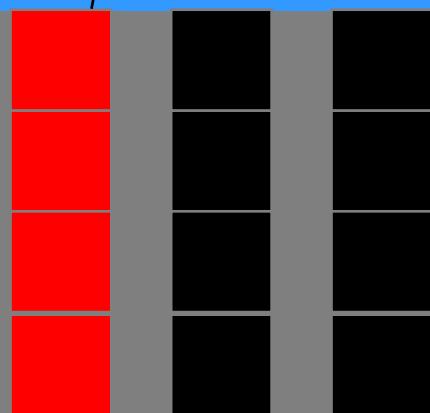
Diophantus

$$4x + 20 = 4$$



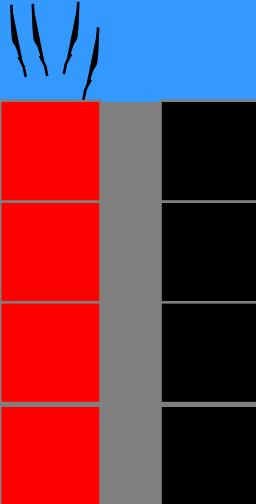
Diophantus

$$4x + 20 = 4$$



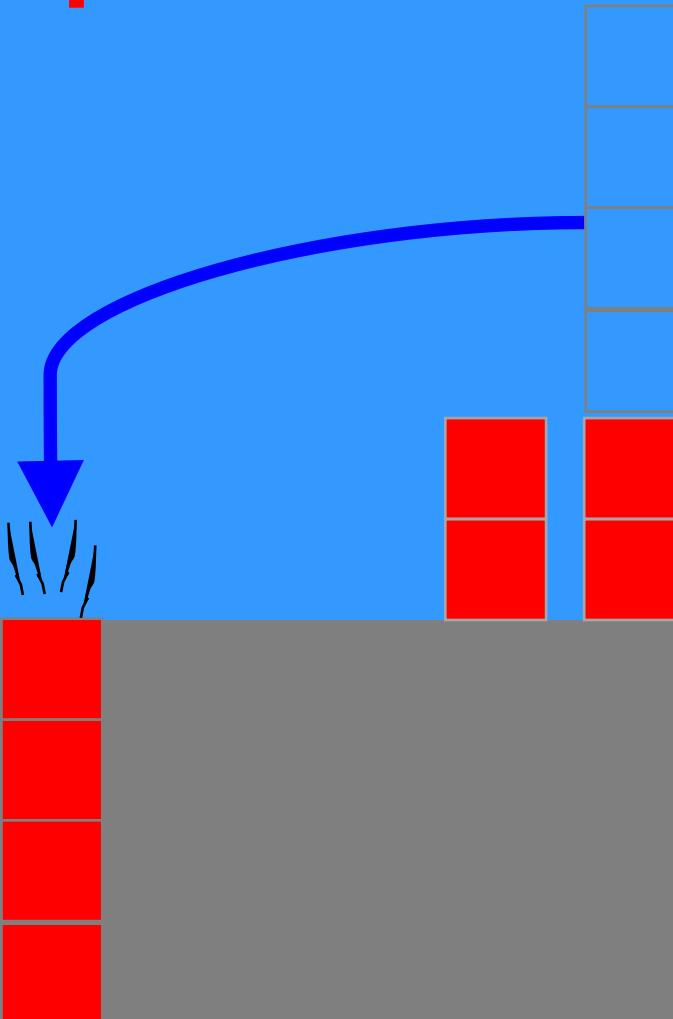
Diophantus

$$4x + 20 = 4$$



Diophantus

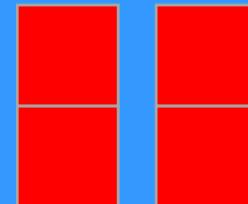
$$4x + 20 = 4$$



Diophantus

$$4x + 20 = 4$$

16 holes + 20 bricks = 4 bricks



Diophantus

$$4x + 20 = 4$$



So, x must be
4 negatives
or -4

Q. WHEN AND WHY DID NEGATIVE NUMBERS FIRST MAKE AN IMPACT IN ENGLAND?

A Description of the
Admirable Table of
Logarithmes
(London, 1616)

A DESCRIPTION OF THE ADMIRABLE TABLE OF LOGA- RITHMES:

WITH

*A Declaration of the most Plenti-
full, Easie, and Speedy vse there-
of in both kinds of Trigonomo-
try, as also in all Ma-
themathicall Calcu-*

Johnnes Braggs Annotations. owner of this book

Inuented and published in Latine by that
Honourable Lord I O H N N E P A I R, Baron of
M A R C H I S T O N, and translated into Eng-
lish by the late learned and famous
Mathematician, EDWARD
W R I G H T.

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W R I G H T.

A. JOHN NAPIER'S
LOGARITHMS 1614
(LATIN) 1616 (ENGLISH)

*But the Logarithmes which
are **lesse then nothing**, we
cal Defectiue, or wanting,
setting this marke - before
them.*

Exp. /Log	+4	+3	+2	+1	0	-1	-2	-3	-4
Number	10,000	1,000	100	10	1	0.1	0.01	0.001	0.0001

The base ten exponent is an absolute value count of the number of times the Unit 1 shifts Left $\times 10 \leftarrow$ or \rightarrow Right $\div 10$

WILLIAM OUGHTRED 1631

And it is to be noted, that a negative Number, how great soever, is less than every both affirmative, and lesser negative.

As -4 is less than 1 , and than -1 .

Also 'tis to be noted, that Subduction changeth the Sign of the Number to be subducted: As out of 4 take 6 , there remains $4 - 6$; that is, -2 . And out of -4 take -6 , their remains $-4 + 6$, that is 2 . Lastly, out of 4 take -6 , their remains $4 + 6$, that is 10 . Wherefore in the extraction of the first single Side, some tryals must be made until you find out the true side; which you shall certainly know by the next grea-

- One of the first major influencers to say negative numbers were less than zero

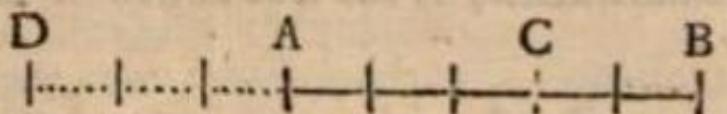


JOHN WALLIS 1685

CHAP.LXVI. Of Negative Squares.

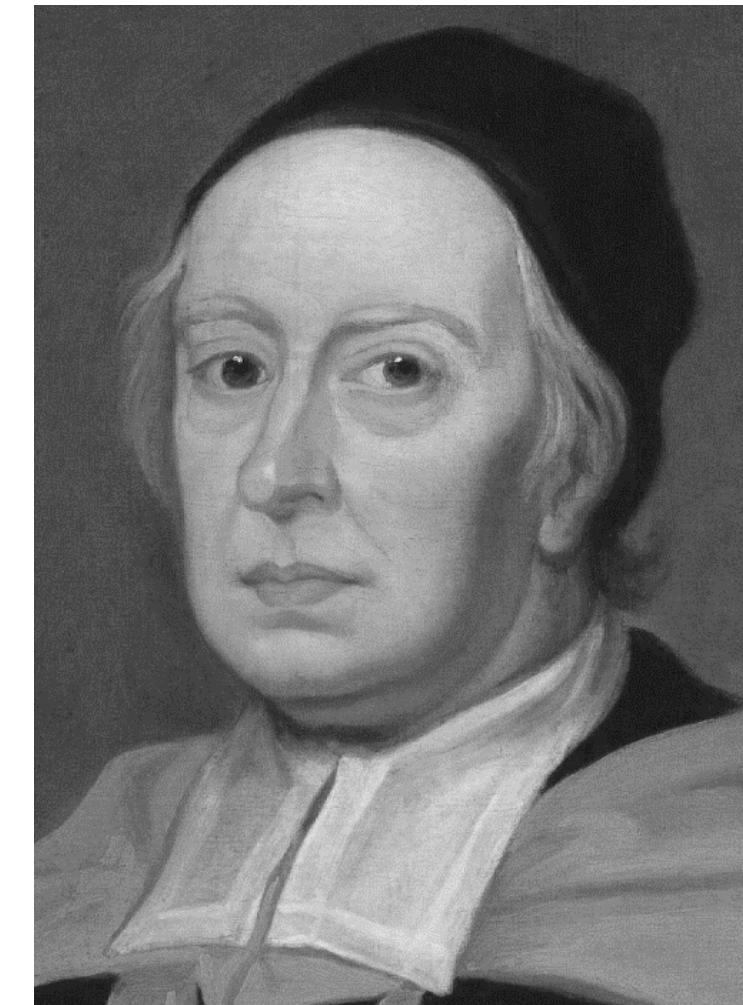
Yet is not that Supposition (of Negative Quantities,) either Unuseful or Absurd ; when rightly understood. And though, as to the bare Algebraick Notation, it import a Quantity less than nothing : Yet, when it comes to a Physical Application, it denotes as Real a Quantity as if the Sign were + ; but to be interpreted in a contrary sence.

As for instance : Supposing a man to have advanced or moved forward, (from A to B,) 5 Yards ; and then to retreat (from B to C) 2 Yards : If it be asked, how much he had Advanced (upon the whole march) when at C? or how many Yards he is now Forwarder than when he was at A? I find (by Subducting 2 from 5,) that he is Advanced 3 Yards. (Because $+5 - 2 = +3$.)



But if, having Advanced 5 Yards to B, he thence Retreat 8 Yards to D; and it be then asked, How much he is Advanced when at D, or how much Forwarder than when he was at A: I say — 3 Yards. (Because $+5 - 8 = -3$.) That is to say, he is advanced 3 Yards less than nothing.

- Negative quantities... import a Quantity less than nothing



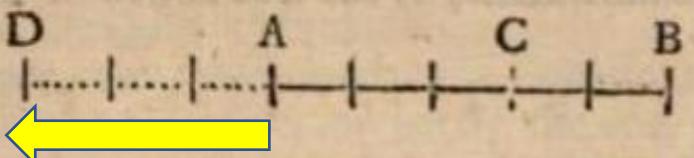
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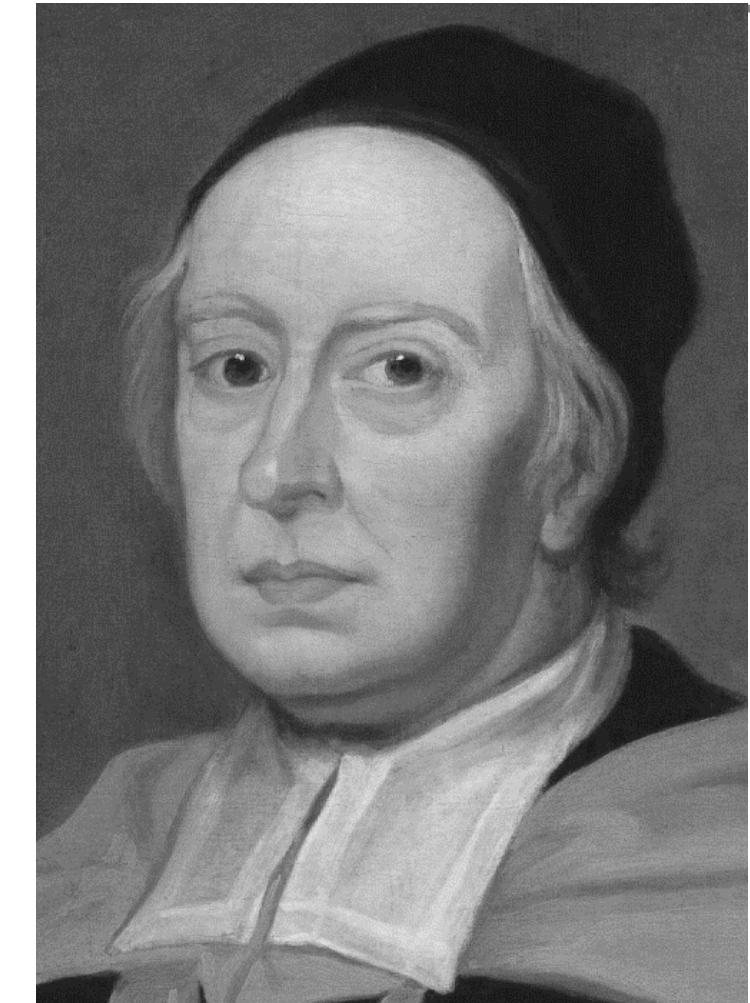
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Less than
nothing



But if, having Advanced 5 Yards to B, he thence Retreat 8 Yards to D; and it be then asked, How much he is Advanced when at D, or how much Forwarder than when he was at A: I say — 3 Yards. (Because $+5 - 8 = -3$.) That is to say, he is advanced 3 Yards less than nothing.

- Presented a line in which a move leftwards 3 yards from A to D was “**less than nothing**”

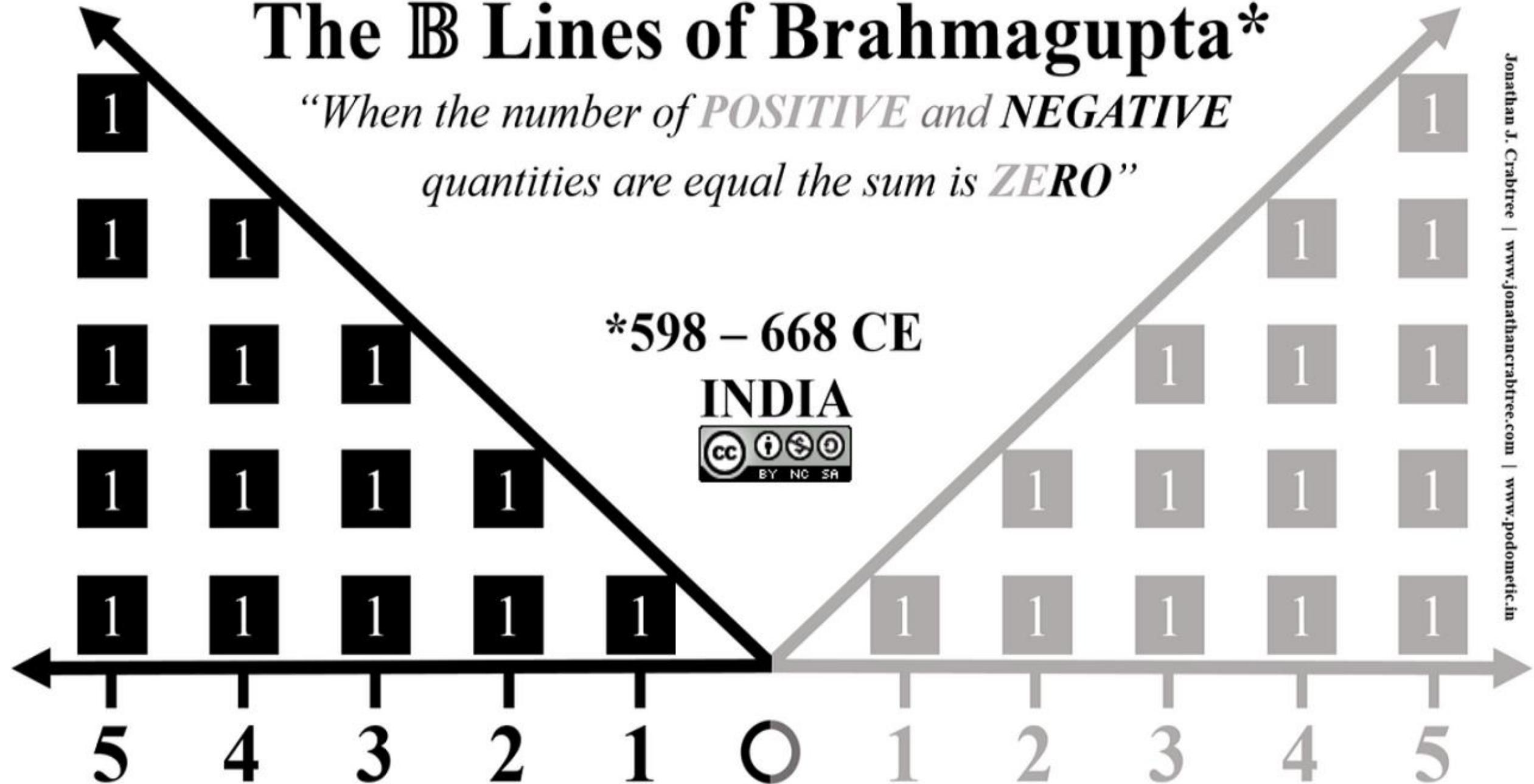


The B Lines of Brahmagupta*

*"When the number of **POSITIVE** and **NEGATIVE** quantities are equal the sum is **ZERO**"*

*598 – 668 CE

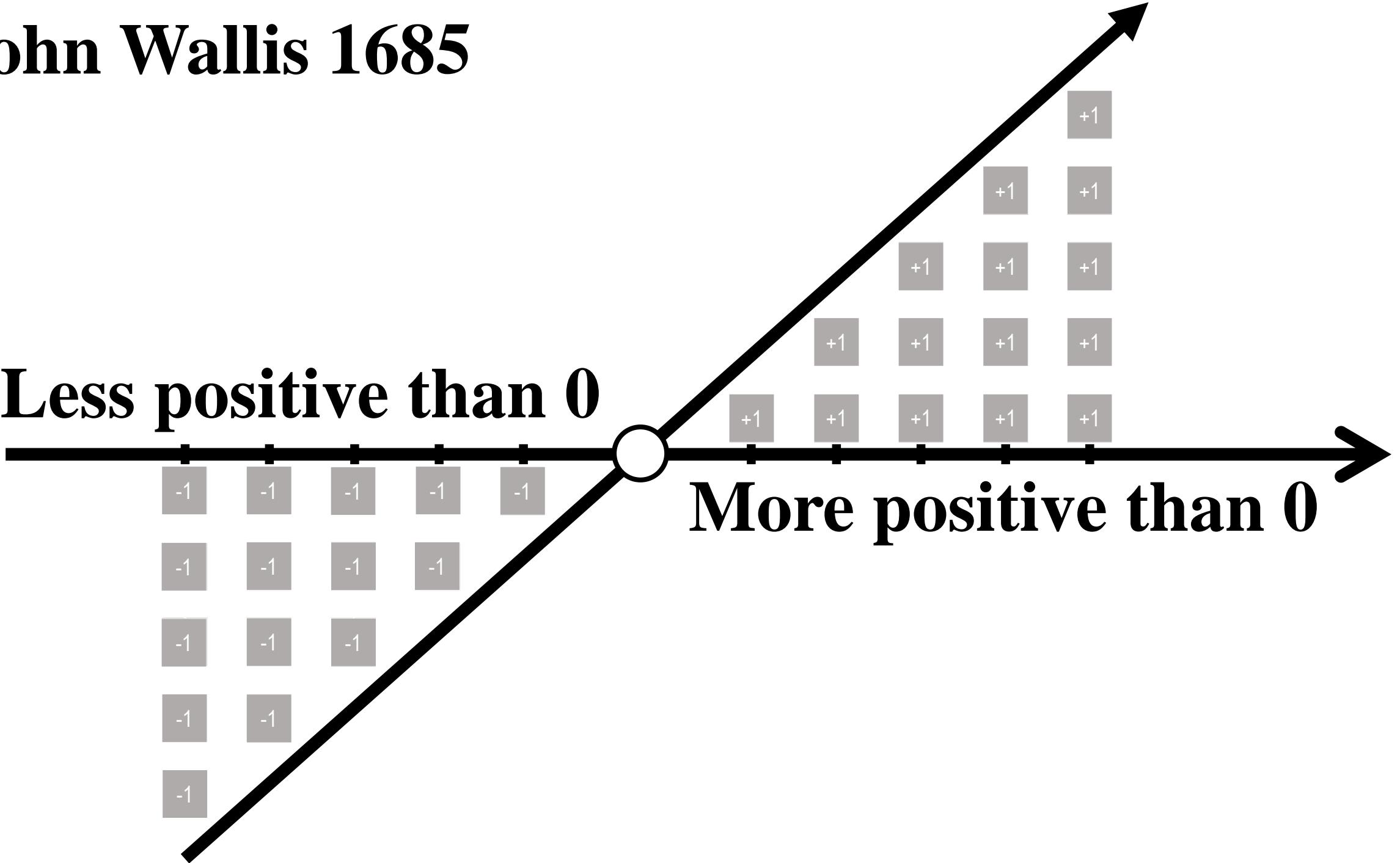
INDIA



John Wallis 1685

Less positive than 0

More positive than 0



Negative Multiplicand ×
Subtracting Multiplier

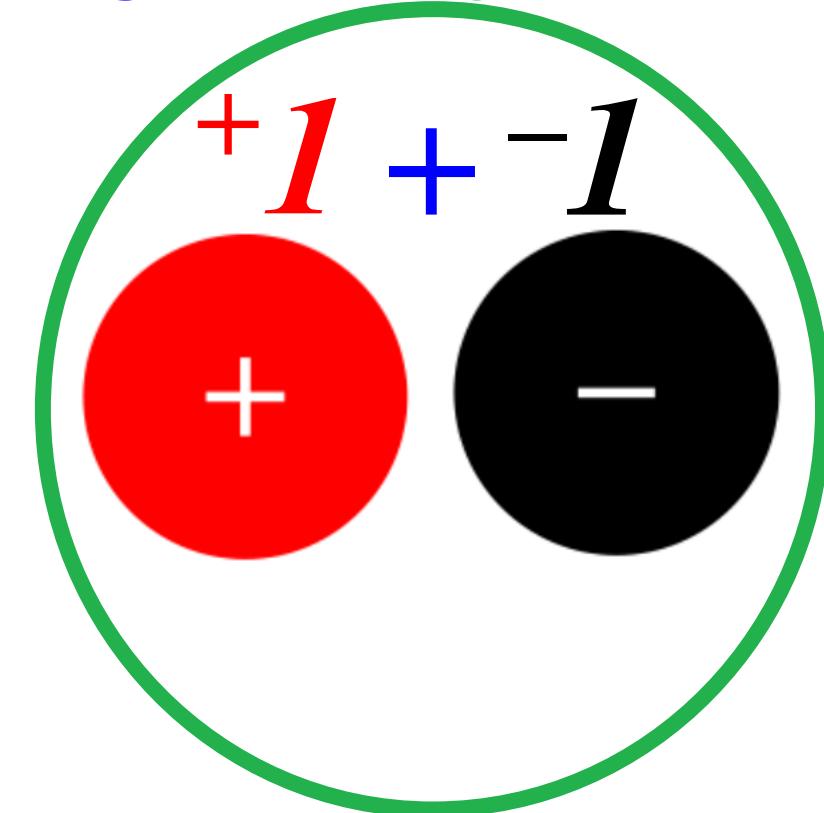
$$-a \times -b$$

$-a$ subtracted from zero b times

$$-1 \times -1$$

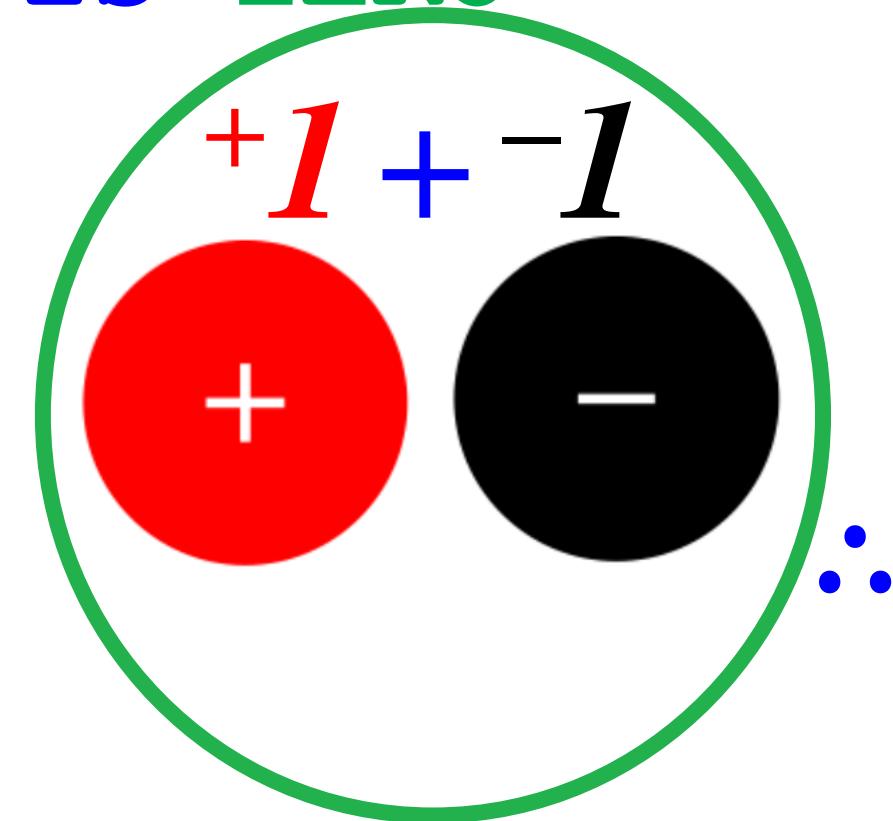
-1 subtracted from zero 1 time

Brahmagupta Defined ZERO
in AS4 when positive and
negative are equal the sum
is ZERO



-1×-1
 -1 subtracted
from zero 1 times

Brahmagupta Defined ZERO
in AS4 when positive and
negative are equal the sum
is ZERO



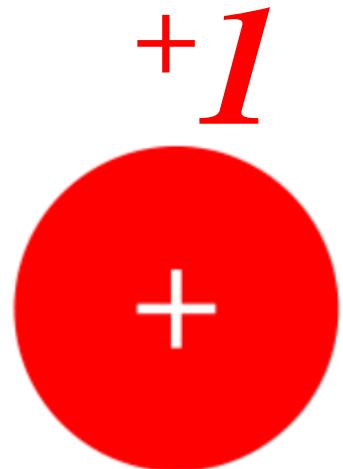
$$\therefore -1 \times -1 = +1$$

-1 subtracted
from zero 1 times
now prove it equals $+1$

Brahmagupta Defined ZERO

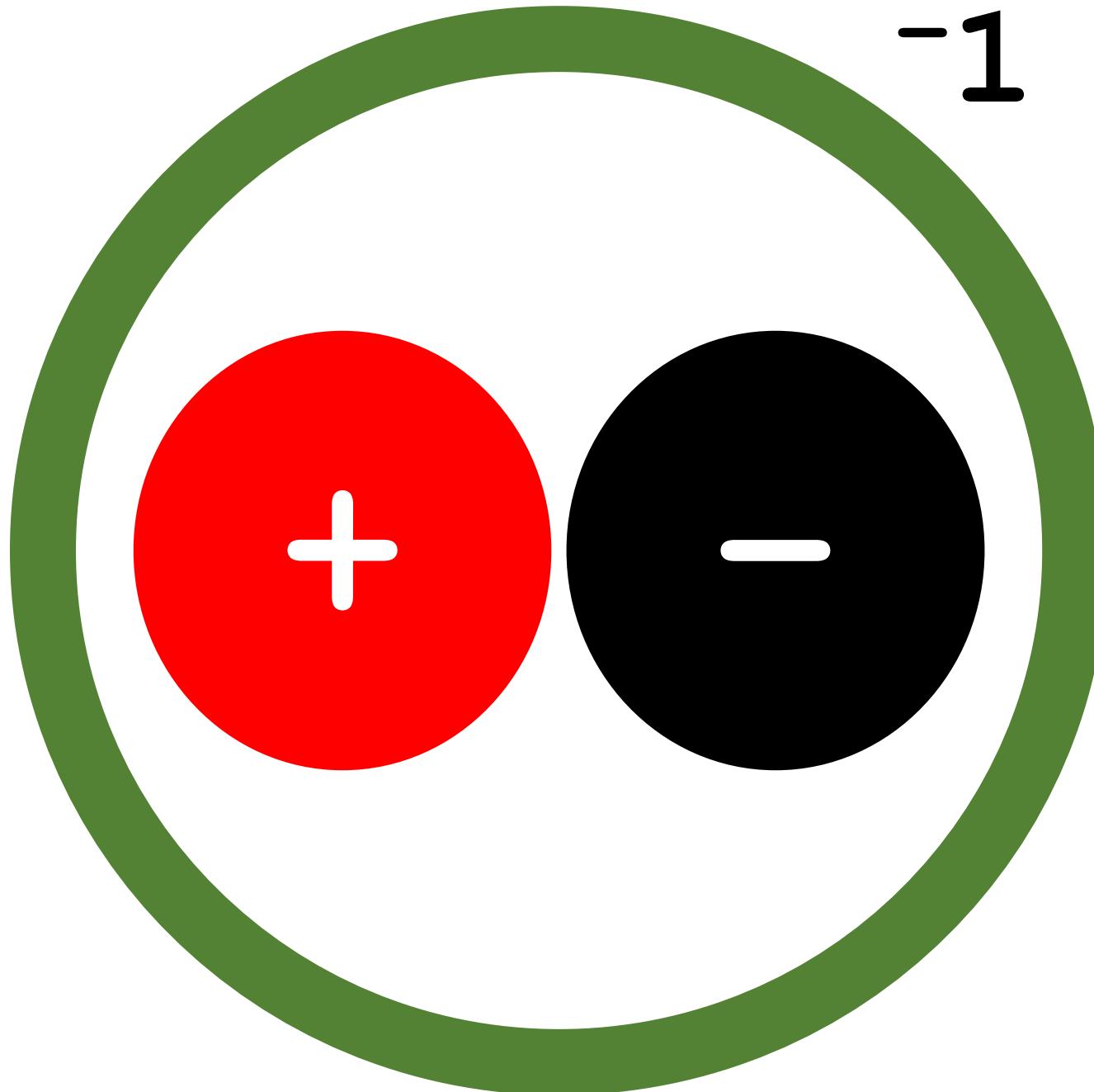
in AS4 when positive and
negative are equal the sum
is ZERO

$$-1 \times -1$$



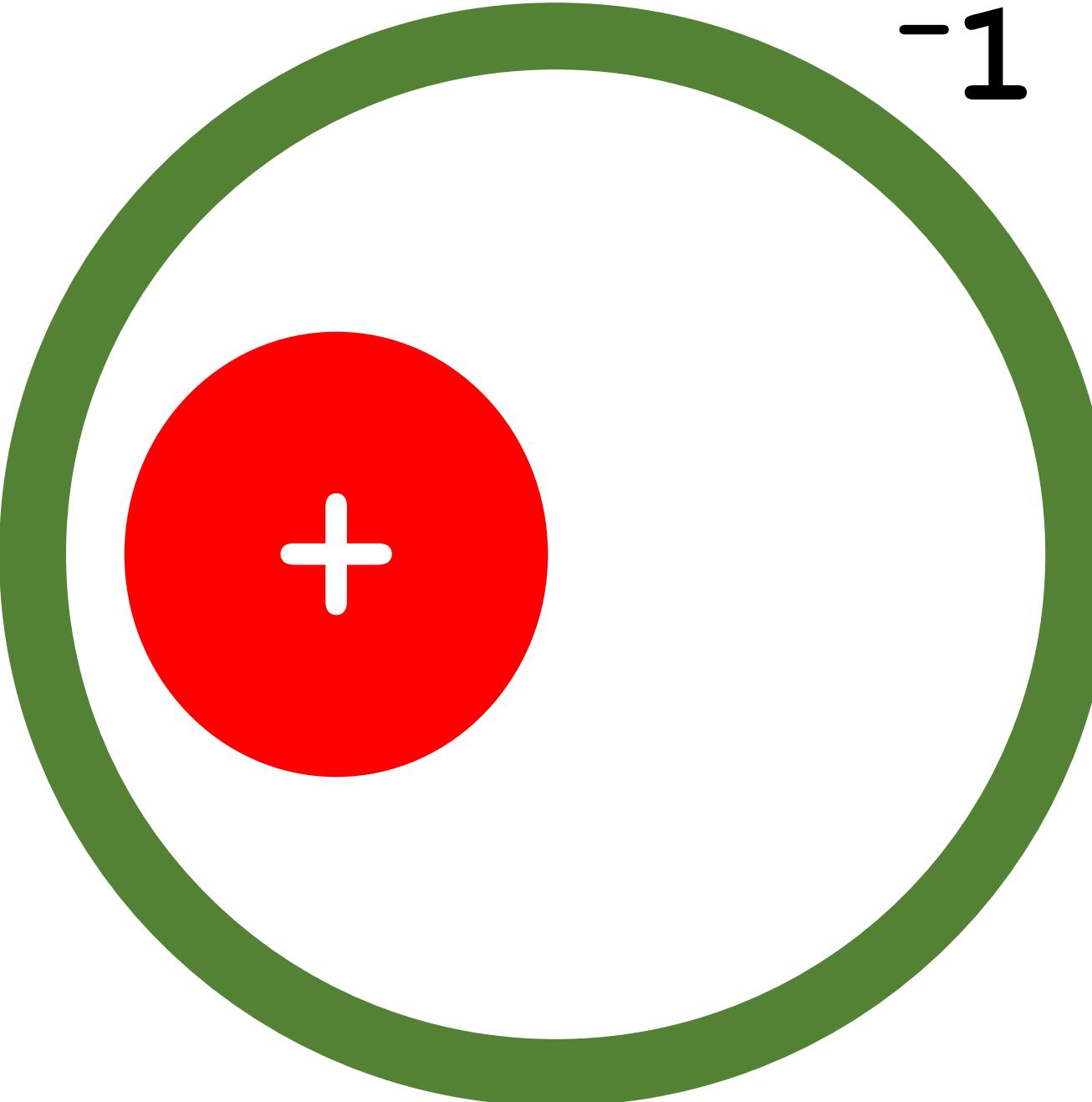
*-1 subtracted
from zero 1 times
now we proved it equals +1*

$$\therefore -1 \times -1 = +1$$



$$-1 \times -1$$

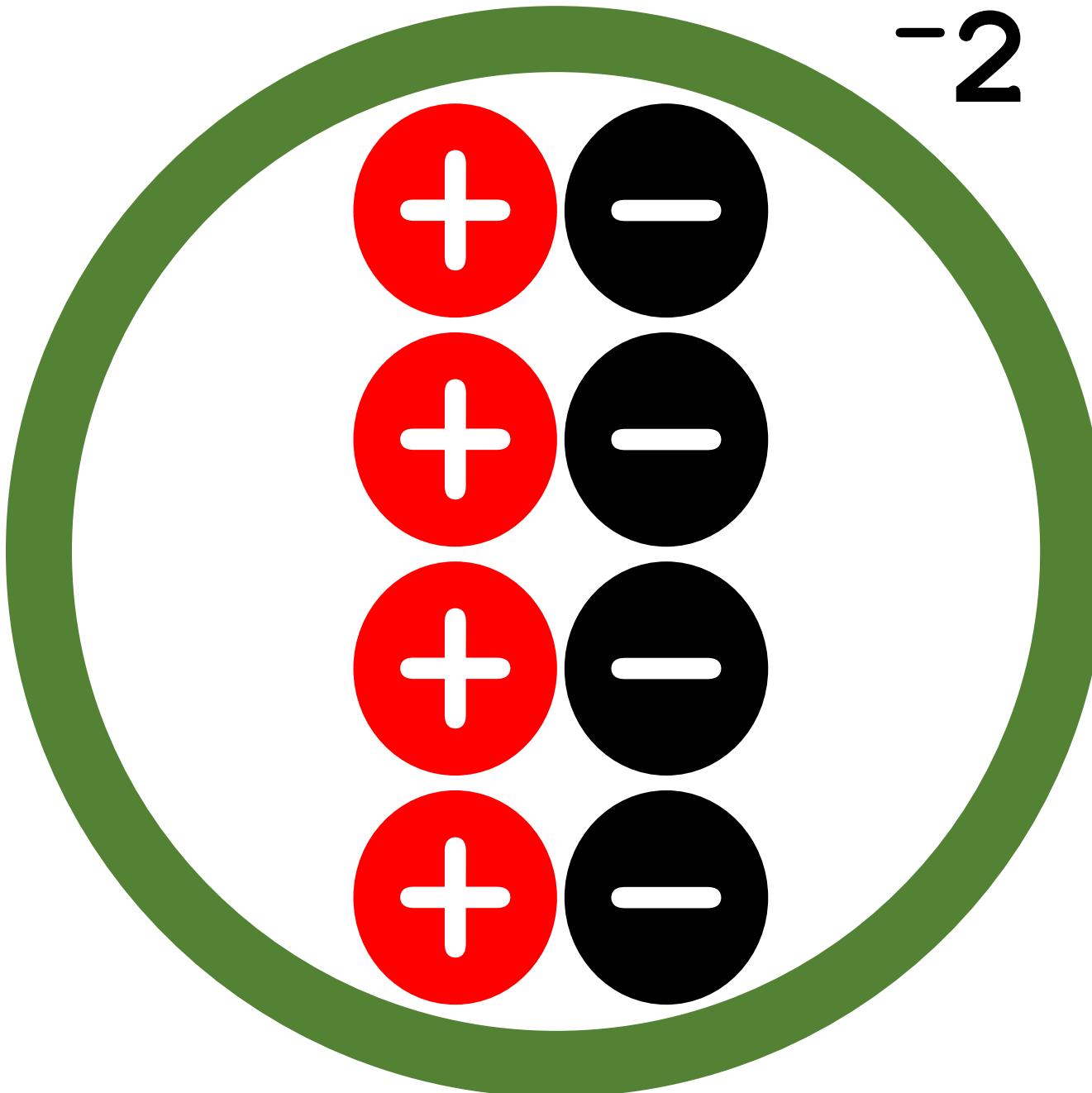
-1 subtracted
from 0 1 time


$$-1 \times -1$$

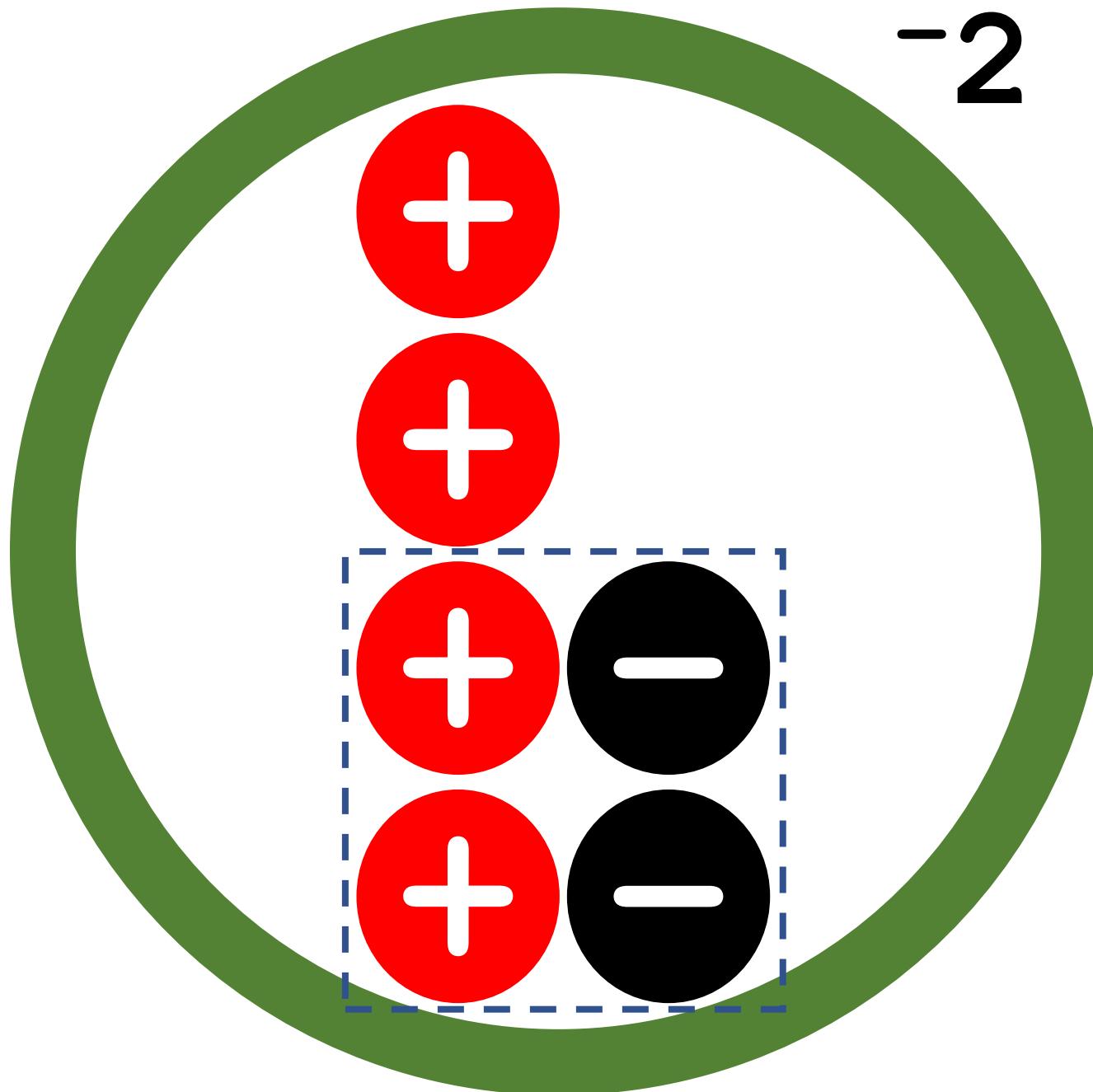
-1 subtracted
from 0 1 time
equals +1

$$-2 \times -2$$

-2 subtracted
from 0 2 times



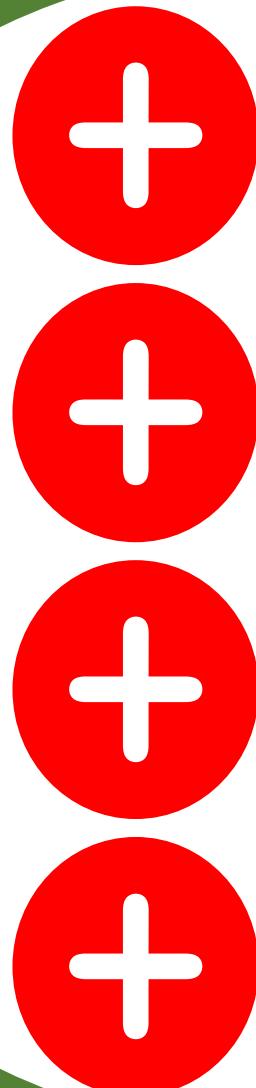
$$-2 \times -2$$



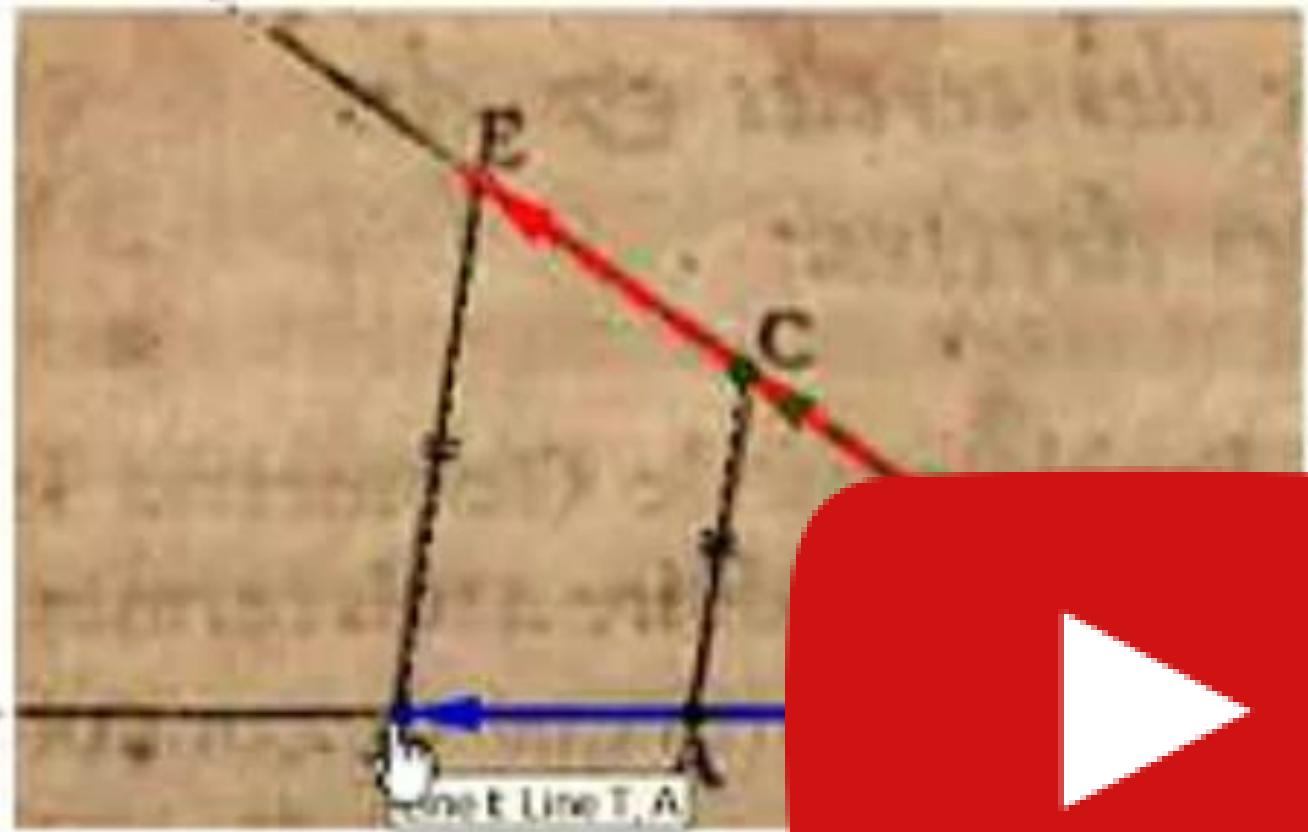
-2 subtracted
from 0 1 time
equals +2

$$-2 \times -2$$

-2 subtracted
from 0 2 times
equals +4



- NOTE: Via the zero-based symmetry of India, we can **build on** the 16th C. writings of Kṛṣṇa Daivajña in his *Bījapallava* translated by **Dr Sita Sundar Ram**.
- Let lines extending in the opposite direction of a positive line segment be negative. Then, we can go back in time to 300 BCE and update Euclid's Book VI Proposition 12!
- We discover a 'new' geometric model for **negative × negative = positive!**



LINE 1 BA = Unit 1



LINE 2 BD = Multiplicand

LINE 3 BC = Multiplier

Join points A & C, and
Draw DE parallel to AC

∴ 4 BE = Product

2:15

See Zero & Negatives Lying Dormant Within
Euclidean Geometry c. 300 BCE

Other Talks

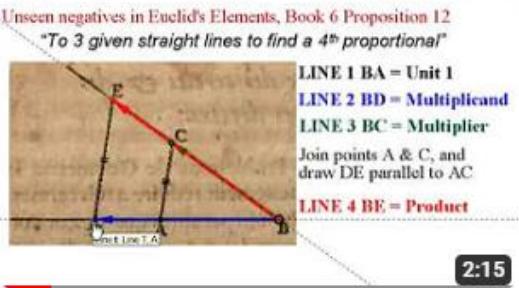
By Jonathan J. Crabtree

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PREMIERING 22 DECEMBER 2021 TO HONOR SRINIVASA RAMANUJAN

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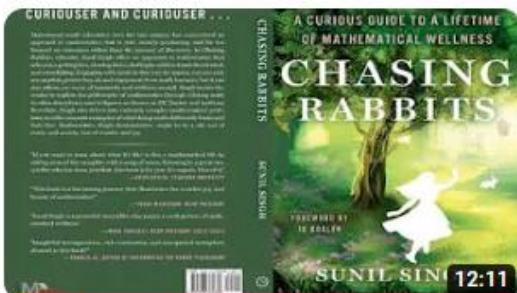
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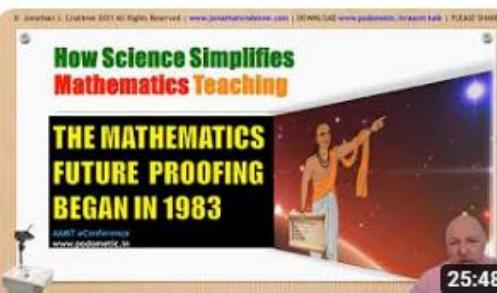
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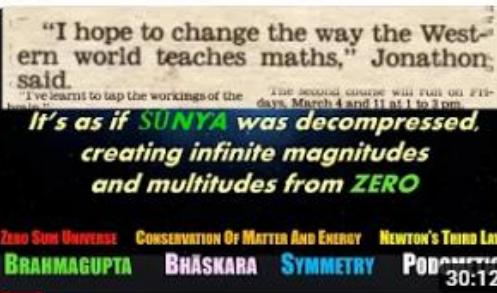
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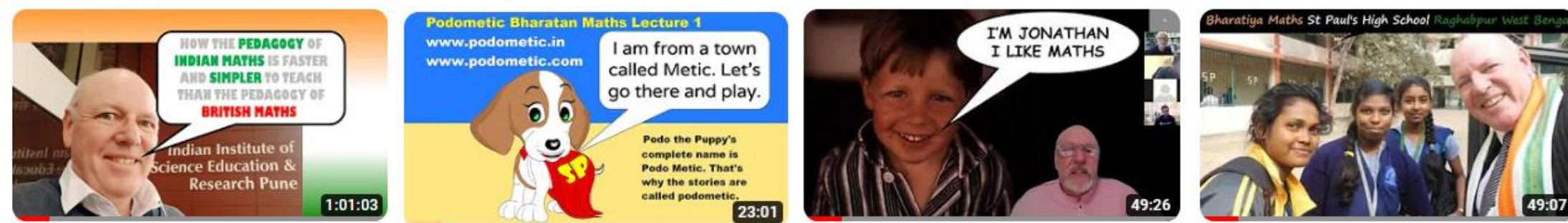
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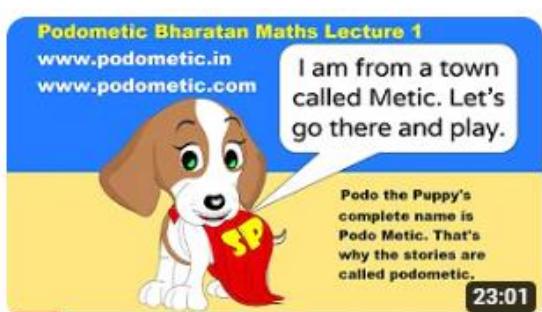


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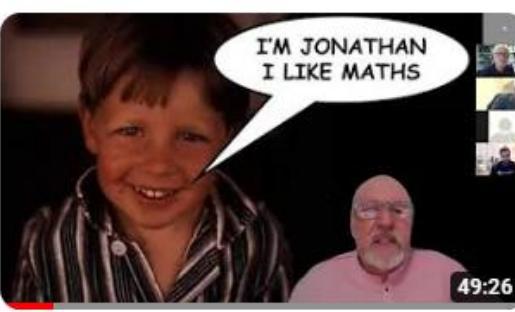
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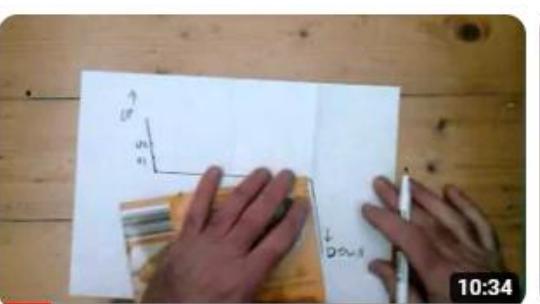
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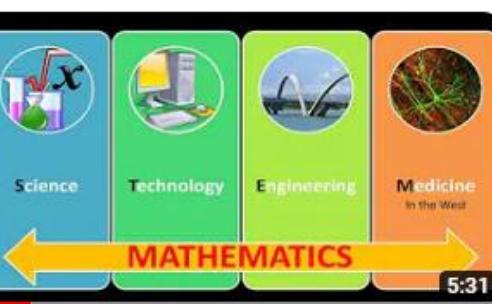
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